

Studies on a Class of Vertical, Submerged and Floating Breakwaters in a Two-Layer Fluid

*A thesis submitted to the
Indian Institute of Technology, Kharagpur
for the award of the degree of*

Doctor of Philosophy

By

P. Suresh Kumar



Department of Ocean Engineering and Naval Architecture

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Kharagpur - 721 302, INDIA

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Dedicated to –

Dr. T. D. Singh (H.H. Bhaktisvarupa Damodara Swami)

my Spiritual Master, for his Encouragements for the Research Work

and

Supreme Personality of Godhead, Lord Jagannath

for His Unlimited Love and Blessings

C E R T I F I C A T E

*This is to certify that the thesis entitled “**STUDIES ON A CLASS OF VERTICAL, SUBMERGED AND FLOATING BREAKWATERS IN A TWO-LAYER FLUID**” being submitted by **Mr. P. Suresh Kumar** to the Indian Institute of Technology, Kharagpur, for the award of the degree of **Doctor of Philosophy**, is a record of bonafide research work carried out by him under our supervision and guidance and that **Mr. P. Suresh Kumar** fulfills the requirement of the regulation of the degree. The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.*

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Dated:

(P. Suresh Kumar)

Preface

In this thesis, a class of problems pertaining to scattering and trapping of harmonic surface- and internal-waves by various types of breakwaters, namely (i) Rigid dikes, (ii) Porous membrane and (iii) Flexible porous plate in a two-layer fluid are studied. These physical problems, under the assumption of the linearized-theory, are reduced to a class of two-dimensional mixed boundary value problems which are then solved for the unknown velocity potentials along with important physical quantities like the reflection and transmission coefficients of an incident time-harmonic wave. Assuming small amplitude response in the cases of flexible structures such as porous membrane and porous plate, the general structural response equation which is coupled with the velocity potentials has been utilized to determine the structural response.

Standard mathematical techniques are utilized, in the reduction and solution of the boundary value problems, such as eigenfunction-expansion method, wide-spacing-approximation method (WSAM) and the least-squares-approximation method.

The content of the thesis is presented in the form of nine chapters. **Chapter 1** is devoted to a general introduction and the objectives of the present study. In **Chapter 2**, an elaborate review of literature and the motivation for the present investigation are presented. The basic mathematical tools utilized in the thesis along with the derivations of the basic hydrodynamics and structural response equations in the linearized set up are elaborated in **Chapter 3**. Although these are available in various text books in a scattered manner, the purpose of this chapter is to present the underlying mathematical formulation in a coherent and connected manner so as to make the thesis self-sufficient.

In **Chapter 4**, surface- and internal-waves scattering by a single surface-piercing rigid

dike is investigated numerically within the context of linearized-theory of water waves. After solving this physical problem the study is extended to a pair of identical rectangular surface-piercing dikes. The surface- and internal-waves scattering by a single bottom-standing rectangular rigid dike and a pair of bottom-standing rigid dikes are presented in **Chapter 5**. In these works, the geometrical symmetry of the problems is being exploited by splitting the velocity potentials into symmetric and antisymmetric components. The solution of the boundary value problem is derived by matched-eigenfunction-expansion method. Because of the flow discontinuity at the interface, the eigenfunctions involved have an integrable singularity at the interface and the orthonormal relation used in the present analysis is a generalization of the classical one corresponding to a single-layer fluid. Computed results in two-layer fluid are compared with those existing in the literature for a single-layer fluid. Moreover, the results obtained by the matched-eigenfunction-expansion method are compared with that of WSAM. The wave reflection characteristics of the system subject to normal incident waves are investigated. The force amplitudes are computed and analyzed for various physical parameters.

In **Chapter 6** and **Chapter 7**, surface- and internal-waves scattering by flexible porous structures is considered. Mathematical models are developed to solve the complex physical problems such as flow past porous flexible structures in a two-layer fluid. In **Chapter 6**, the scattering of water waves by a flexible porous membrane breakwater in a two-layer fluid having a free surface is analyzed in two-dimensions. Linear wave theory and small amplitude membrane response is assumed. The porous-effect parameter used in the study is a complex number, which includes both inertia and resistance effects. The porous membrane breakwater is tensioned and pinned at both the free surface and the seabed. The associated mixed boundary value problem is reduced to a linear system of equations by utilizing a more general orthogonal relation along with least-squares-approximation method. The reflection and transmission coefficients for the surface- and internal-waves, free surface and interface elevations, and the non-dimensional membrane deflection are computed for various physical parameters like the non-dimensional tension parameter, porous-effect parameter, fluid density ratio, ratio of water depths of the two

fluids, to analyze the efficiency of a porous membrane as a wave barrier in the two-layer fluid. The problem is then extended to the case of a flexible plate in **Chapter 7**. The plate breakwater is extended over the entire water depth and the problem is analyzed in two-dimensions. The reflection and transmission coefficients for the surface- and internal modes, wave load and breakwater response are computed for various physical parameters of interest to analyze the efficiency of the flexible porous plate as a breakwater in the two-layer fluid.

In **Chapter 8** the surface- and internal-wave trapping by porous and flexible partial breakwaters near the end of a semi-infinitely long channel is studied in two-dimension. In the study, both surface-piercing and bottom-standing configurations are considered. The surface-piercing breakwater is clamped above the free surface and is free at the other end which is submerged in the fluid. On the other hand, the bottom-standing breakwater is fixed at the seabed and the other end is having a free edge. A combination of eigenfunction-expansion method and least-squares-approximation method is used to solve the associated mixed boundary value problems. The reflection coefficients are obtained and discussed for both surface- and internal-waves for different values of non-dimensional fluid density ratio, porous-effect parameter, normalized distance between breakwater and channel end-wall, length of submergence and flexural rigidity of the breakwaters for both surface-piercing and bottom-standing cases. Furthermore, the hydrodynamic force acting on the flexible partial breakwaters and the breakwater response are determined for different physical parameters of interest.

Chapter 9 provides concluding remarks on the study. The scope of the present research work is also described followed by future scope of work. **Bibliography** has been included separately.

Keywords:

Surface-waves; Internal-waves; Wave reflection; Wave transmission; Wave scattering; Wave trapping; Resonance; Dissipation; Dikes; Flexible breakwaters; Porous breakwaters; Surface-piercing breakwaters; Bottom-standing breakwaters; Eigenfunction-expansion method; Least-squares-approximation; Wide-spacing-approximation; Orthogonal relation;

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List of Notations

a	half of the dike width
b	thickness of the porous medium
C_M	added mass coefficient of medium grains
d	flexural rigidity
f	resistance force coefficient
Fr	hydrodynamic force
G	porous-effect parameter
g	acceleration due to gravity
H	total depth of entire fluid domain
h	depth of upper fluid
I_0	amplitude of incident wave
I_I, I_{II}	amplitude of incident wave in SM and IM
K	$= \omega^2/g$
Kr_I, Kr_{II}	reflection coefficient in surface mode (SM) and internal mode (IM)
Kr_I, Kr_{II}	transmission coefficient in SM and IM
k_0	incident wave number
L_g	vertical gap location
L_{lf}, L_{uf}	portion of breakwater in lower and upper fluid
L_{op}	portion of breakwater above free-surface
m_s	flexible breakwater mass
m'	non-dimensional membrane mass
n_i	outward normal to the boundary

p_n	roots of dispersion relation
pr	fluid pressure
R_I, R_{II}	amplitude of reflected wave in SM and IM
s	two-layer fluid density ratio
S	inertia force coefficient
T	tension applied to the membrane
T_I, T_{II}	amplitude of transmitted wave in SM and IM
T'	non-dimensional tension parameter
U, V	horizontal and vertical fluid velocity
x, y	horizontal and vertical Cartesian co-ordinates
β	breakwater frequency parameter
γ	porosity
ζ	breakwater deflection (function of y and t)
η_{fs}, η_{int}	free surface and interface elevation
λ_I	wave length of incident wave in SM
ξ	breakwater deflection (function of y)
ρ_1, ρ_2	density of upper and lower fluid
ρ_s	membrane mass density
Φ	velocity potential (function of both <i>space</i> and <i>time</i>)
ϕ	spatial velocity potential
ω	wave frequency

List of Abbreviations

BIE	boundary integral equation
DBC	dynamic boundary condition
HF	horizontal force
IM	internal mode
IPBO	interface piercing bottom obstacle
IPSO	interface piercing surface obstacle
KBC	kinematic boundary condition
SM	surface mode
VF	vertical force
VLFS	very large floating structure
WSAM	wide-spacing-approximation method

Chapter 1

INTRODUCTION

Water waves are generated mainly by winds in open seas and large lakes. They carry a significant amount of energy from winds into near-shore regions. Thereby they significantly contribute to the regional hydrodynamics and transport process, producing strong physical, geological and environmental impact on coastal environment and on human activities in the coastal area. Furthermore waves are closely connected to the complete dynamics of coastal systems.

Understanding wave dynamics and predicting wave conditions in harbor and coastal regions remained an important part of coastal studies over the years. It is particularly critical to coastal engineering practice in the aspects of, for instance, shoreline protection, beach erosion, navigation safety, channel maintenance, harbor planning and management, breakwater and jetty design, etc. Moreover an accurate prediction of the hydrodynamic effects due to waves interaction with offshore structures is a necessary requirement in the design, protection and operation of such structures.

1.1 TWO-LAYER FLUID

Water waves have been studied for more than three centuries. Since the pioneering work of Isaac Newton (1687), it is long known that an exact analytical solution of the water wave problem in general is out of reach. The study and theories dealing with ordinary

surface gravity waves are well known and enough information is available on ordinary surface gravity waves in the open literature. The theories on the water wave propagation have been developing over the years, and research interest in the recent past appears to be shifting towards solution of more practical real field problems.

In the recent time, it is observed that there is an increasing interest in understanding internal-waves because often in an ocean, internal-waves are observed and are the cause of heavy damages experienced by many onshore and offshore structures. For ocean engineers, interest in internal-waves is due to their role in submarine detection and the generation of anomalous drag on ships in fjords and estuaries. Such anomalous drag occurs when fresh water from rivers and runoff forms a thin layer of light fluid which lies above the cold saline water in a narrow fjord. The passage of a ship can then generate internal-waves which radiate energy away from the neighborhood of the ship. This lost energy is an additional wave drag for the ship. Earlier the source of this energy loss was not easy to identify and was regarded as mysterious. Regions having this drag came to be known as “dead water” regions.

There is one well known theory that the loss of the American nuclear submarine, “Thresher,” in the early 1960s was a result of the effects of a collision with an internal wave. Internal waves are not only dangerous to submarines, but also can have disastrous effects on drilling platforms, piers and viaducts. Internal tides cause dramatic vertical displacements of density surfaces in the ocean interior, often several tens of meters and even hundreds of meters in some locations (Garrett and Kunze (2007)). These displacements, and the associated currents, complicate the mapping of the average state of the ocean, and also have major effects on acoustic transmission (Dushaw 2006), sediment transport (Cacchione et al. (2002) and McPhee-Shaw (2006)), and oil-drilling platforms (Osborne and Burch 1980, and Osborne et al. (1978)).

Lowering of the salinity of surface waters of the Ocean near a river outflow or thawed glacier causes deceleration of a ship because internal waves may be generated above the interface between the low-salinity surface and steeper layers of salty waters. The appearance of “dead water” can have an influence on the maneuvering of submarines. Sailboats

and towed vessels are often thrown by this phenomenon. The more general scientific interest comes from the general role of internal-waves in energy transport in lakes and oceans. Moreover, the major challenge in case of internal-waves is that they are not visible to the naked eye, hence it is difficult to detect them and take precautionary measures.

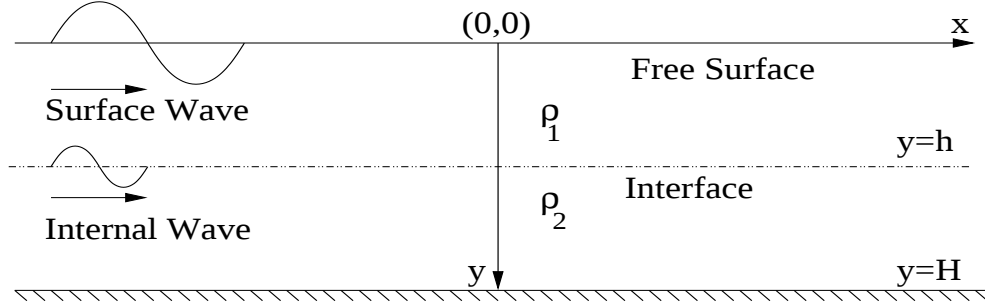


Figure 1.1: Definition sketch for two-layer fluid wave motion.

The simplest model of an internal-wave is when the density of the liquid is taken to be piecewise constant. This means, the liquid can be described as two-layers of constant density over each layer. In many natural bodies of water, stratification of either temperature or salinity may take place which can lead to significant density differences with depth. A sharp change in the fluid density at a certain water depth owing to variation in salinity and/or temperature may be observed in a lake, an estuary or Norwegian fjords. Another example of sharp density change is a thin layer of muddy water at the bottom of harbors or channels with relatively shallow water depth. These density changes may significantly alter the hydrodynamic characteristics of waves past coastal structures. In the present thesis, attention is restricted to the typical case of an ocean, fjord, or lake where the upper layer (marked as Fluid 1 in the Fig. 1.1) is a relatively light liquid, and the lower layer (marked as Fluid 2 in Fig. 1.1) is a relatively heavy liquid. In such stratified flows a new type of wave can be observed due to the new mode known as internal-wave.

Two distinct wave modes are possible for the two-layer fluid comprising of a light liquid, and a heavy liquid. These two modes are known as the surface or fast mode and the internal or slow mode. The surface or fast mode gives rise to motions which are similar to those produced by an ordinary surface-wave. Because of the similarity of the mode shapes, it should not be surprising that the wave speed of the surface or fast mode usually

differs very little from the usual gravity wave speed. Internal-waves are generated due to periodic movements of interface. This motion is usually not observable at the surface to the naked eyes since it may not effect the surface. The restoring force for internal-waves is proportional to the product of gravity and the density difference between the two layers (the relative buoyancy). At internal interfaces this difference is much smaller than the density difference between air and water (by several orders of magnitude). As a consequence, internal-waves can attain much larger amplitudes than surface-waves. It also takes longer for the restoring force to return particles to their average position, and internal-waves have periods much longer than surface gravity waves. Hence, the speed of the internal-wave is considerably smaller than the speed of the surface-wave.

In contrast to surface-waves in which horizontal particle velocities are largest at the surface and either decay quickly with depth (in deep water waves) or are independent of depth (in shallow water waves), horizontal water movement in internal-waves is largest near the surface and bottom and minimal at mid-depth. A second distinctive feature of internal-waves is that the surface disturbance can be extremely small relative to the size of the disturbances to the internal layer. Even though understanding of such interesting physical phenomenon in an ocean is very much important in the coastal engineering applications, very little knowledge on this is available up to date because of the complexity involved in mathematical formulation and understanding the physics of internal-waves which propagate simultaneously with the surface-waves in a two-layer fluid.

1.2 WAVE-STRUCTURE INTERACTION

In general wave-structure interaction can be separated into hydraulic responses (such as wave run-up, wave over-topping, wave transmission and wave reflection), and loads and response of structural parts. Design conditions for coastal structures includes acceptable levels of hydraulic responses in terms of wave run-up, over-topping, wave transmission and wave reflection.

The wave run-up level is one of the most important factors affecting the design of

coastal structures because it determines the design crest level of the structures in cases where no (or marginal) over-topping is accepted. Examples includes dikes, revetments and breakwaters with pedestrian traffic. Wave over-topping occurs when the structure crest height is smaller than the run-up level. The over-topping discharge is a very important design parameter because it determines the crest level and the design of the upper part of the structure. Design levels of over-topping discharges frequently vary, from heavy over-topping of detached breakwaters and outer breakwaters without access roads, to very limited over-topping in case where roads, storage areas and moorings are close to the front of the structure.

With partial breakwaters the incident waves will more or less pass over or under the structure while retaining much of the incident wave characteristics. In case of impermeable flexible structures, wave transmission takes place when the impact of water waves generates new waves at the rear side of the structure because of the structural deformation. Permeable structures allow wave transmission as a result of wave penetration.

Design levels of transmitted waves depend on the use of the protected area. Related to port engineering is the question of acceptable wave disturbance in harbor basins, which in turn is related to the movements of moored vessels. Coastal structures like breakwaters reflect some portion of the incident wave energy. If reflection is significant, the interaction of incident and reflected waves can create an extremely complex sea with very steep waves that often are breaking. This is a difficult problem for many harbor entrance areas where steep waves can cause considerable maneuvering problems for smaller vessels. Strong reflection also increases the seabed erosion potential in front of protective structures. Hence wave reflection from the boundary structures like breakwaters determines to a large extent the wave disturbance in harbor basins and maneuvering conditions at harbor entrances. Moreover, breakwaters can cause reflection of waves onto neighboring beaches and thereby increase wave impacts on beach processes. In addition, an important part of the design procedure for structures in general is the determination of the loads and the related stress, deformations and stability conditions of the structural members.

1.3 BREAKWATERS

The destructive power of ocean waves is well known and the methods to provide protection against these waves have occupied the attention of coastal engineers over the years. Waves moving over the ocean near the shore may greatly change beaches. Strong winds often creates a situation, which raises the water level and exposes higher parts of beach to wave attack which are not ordinarily vulnerable to waves. These waves carry away large quantity of sand from the beach to the near shore bottom. Land structures, inadequately protected and located too close to the water, are then subjected to the hydrodynamic forces of waves and may be damaged and destroyed. Recent rise in damage of both life and property across the shore lines due to the destructive ocean waves all over the world further intensified the research to find suitable protection techniques against various type of wave attacks.

The term wave transmission is used in reference to the wave energy that travels past a breakwater, either by passing through and/or by over-topping the structure. The wave energy that is attenuated in the lee of the breakwater is either dissipated by the structure (due to friction, wave breaking, armor unit movement, etc.) or reflected back as reflected wave energy. The effectiveness of a breakwater in attenuating wave energy can be measured by the amount of wave energy that is transmitted past the structure: lesser the wave transmission coefficient the greater the wave attenuation. Breakwaters are designed not only to protect the offshore and onshore structures from wave attack but also used to provide economic protection to harbor, marine and restore eroding beaches.

The type of protection needed is a function of the purpose it has to serve. In general, natural or artificial breakwaters of various configurations are used depending on the applications to provide protection from wave attack in both onshore and offshore areas. Breakwaters are generally shore-parallel structures and their primary purpose is to reduce the amount of wave energy reaching the protected area. They are similar to natural bars, reefs or near-shore islands and are designed to dissipate wave energy. Beaches and dunes can be protected by an offshore breakwater that reduces the wave energy reaching the

shore. However, offshore structures are usually more costly than onshore structures and are seldom built solely for shore protection. Offshore breakwaters are constructed mainly for navigation purpose. A breakwater protecting a harbor area provides shelter for ships and boats.

Over the years, investigations have been carried out on various types of breakwaters to find the efficient breakwater suiting a particular application and the coastal region that needs protection. The major challenge encountered is to select the proper breakwater configuration, material and location so that it will be effective, economical and environment friendly. For example, in situations where harbors are located in areas where severe wave conditions occur they often lie sheltered behind one or more breakwaters. The location of the breakwater should be chosen such that the ships in the harbor will be subjected to a gentle wave climate.

1.3.1 Rigid Breakwaters

The rigid-fixed breakwaters offer advantage in the form of excellent storm protection. However, at the same time they contribute several drawbacks to the environment. Careful thought must be given to the fixed breakwaters: they become an almost permanent part of the landscape. Hence, if any environmental damage they may cause must either be accepted or the breakwater must be removed. This may be a very expensive penalty for a mistake. A rigid-fixed breakwater must not only be carefully designed, but also very carefully analyzed for its effects on the physical system in which it is to be placed.

Another disadvantage is that a fixed breakwater can be a total barrier to close off a significant portion of a waterway or entrance channel, thereby causing a faster river or tidal flow in its vicinity, as well as potentially trapping debris on the up-drift side. It may create unacceptable sedimentation and water quality problems due to poor circulation behind the structure. On the other hand, a detached breakwater may be connected to the shore by the formation of a tombolo. This could seriously interrupt long-shore transport and cause down-drift erosion. Although most existing breakwaters are rigid structures that are fixed to the ocean floor and projected above the water surface, the shift has

been towards more temporary, transportable breakwaters because of the aforementioned reasons.

1.3.2 Flexible Breakwaters

Recent developments in materials science have contributed significantly to opening an opportunity for the flexible structures as breakwaters. The problem of material degradation and failure have been largely overcome by modern materials. With the use of modern materials, flexible structures might be extremely effective as breakwater, absorbing or reflecting much of the wave energy. There has been an increasing interest in the use of flexible breakwater as a means of providing protection from wave attack in semi-protected regions. Furthermore, flexible breakwaters have generated interest among researchers with some of its added advantages.

Such structures provide an alternative to more conventional rigid-fixed breakwaters in areas where poor foundation conditions exist or where protection is required only on a temporary basis. They can be utilized as temporary sacrificial breakwaters to reduce the size of storm waves impacting harbors, coastal areas or fixed breakwaters. Another advantage is that flexible breakwaters could provide an inexpensive means of protecting beaches and shorelines exposed to small or moderate waves and offer fast and relatively easy installation for temporary offshore work. They are reusable, have a lower construction cost and are lighter in weight compared to the conventional large massive structures. They can also be more economical compared to fixed-type breakwater especially when the water depth is large. This is because their cost does not increase substantially with depth, while the cost of a fixed structure, such as rubble-mound breakwater, increases exponentially with depth.

Furthermore, flexible structure can be prefabricated onshore thus reducing construction time and complexity. They can be moored despite unfavorable seabed soil conditions and are usually positioned near the water surface where the wave energy is most pronounced. Flexible structures are also attractive because they require only a little maintenance, they can handle extreme temperature and they usually do not corrode. Because of

the aforementioned advantages, flexible breakwaters can support a number of operations at sea, such as oil and gas extraction, fish farming, ocean mining and recreation. These structures can also be used for pollution control, salvage operations, and construction and maintenance of offshore platform. They can serve as a breakwater augmentation device by reducing the size of waves incident on fixed breakwaters, shores and offshore structures.

In past flexible plate as a breakwater were more popular (Stoker (1957), Siew and Hurley (1977), Patarapanich (1978), Cheong and Patarapanich (1992), and Wang and Shen (1999)). However, in recent studies flexible wave barrier consisting of a vertically tensioned membrane have been reported (Kim and Kee (1996), Kee and Kim (1997), Lo (1998) and Lo (2000)). These refer to structures that mainly consist of membrane made of synthetic fiber, rubber or a polymeric material. They can be easily fabricated in large size, allowing for wave control in a wide region. The multi-mode motions of the membrane can also be explored to widen the effective frequency range for wave attenuation, especially on irregular waves.

1.3.3 Porous Breakwaters

Impermeable breakwaters have been employed to block waves in harbor for a fairly long time. Their design is simple and convenient. When they are subjected to wave impact, the wave load is extremely high because there is almost a total reflection of wave energy. It has been well-known for a long time that a harbor may be agitated into a resonant state when subjected to incoming waves of a particular period. Such resonance may lead to extremely large oscillations of the water surface within the harbor and cause disastrous damage to its mooring system, especially when the breakwater is a rigid structure. Therefore, care must be taken in the design to avoid resonance in the harbor. In the case of possible resonance, wave energy must be radiated or dissipated so that the oscillation can be effectively suppressed. Recently porous breakwaters are proposed to tackle such difficult practical problems.

Porous breakwaters can reduce both the transmitted and reflected wave heights. This kind of structures also experience less hydrodynamic forces and dissipate a significant

amount of wave energy. In general flow or wave motion past porous media has strong relevance in many distinct fields, such as soil mechanics, biology, hydrology and harbor engineering. It has both academic and practical importance. However, a through understanding of the problem is still far from being complete, even though its applications in coastal engineering and harbor engineering is extremely common. Some of the major applications of porous breakwaters are oil/contaminant spill containment, temporary protection during coastal construction works and augmentation of existing breakwaters for seasonal protection. The ability of an engineer to predict wave transmission and reflection by a porous breakwater plays a central role in the protection of both onshore and offshore structures. The knowledge about stability of the breakwater under wave attack is also valuable to harbor engineering.

1.3.4 Partial Breakwaters

Most of the existing breakwaters are in general extended over the entire water depth, i.e. extended from the seabed up to the free surface. However, in recent years, there is a significant interest in the use of partial breakwaters to attenuate the wave energy. Partial breakwaters only occupy a segment of the whole water depth. In coastal engineering, partial barriers as breakwaters are more economical and sometimes more appropriate for engineering applications. These kinds of breakwaters also provide a less expensive means to protect beaches exposed to waves of small or moderate amplitudes, and to reduce the wave amplitude at resonance. These breakwaters can either be suspended or mounted to the ocean floor depending on the application and economic constraints. For the first case they can be floating or partially submerged and are known as surface-piercing breakwaters. For the second case they are usually fully-submerged and are known as bottom-standing breakwaters.

Surface-piercing breakwaters are in general temporary and transportable breakwaters. They are economical alternative to fixed structures for use in deeper waters (at depth greater than 20 feet) and can effectively attenuate moderate wave heights (less than about 6.5 feet). Poor soil conditions may make surface-piercing breakwaters the

only options available. They also minimize the interference on water circulation and fish migration. Further if ice formation presents a problem, surface-piercing breakwaters can be removed from the site. Surface-Piercing breakwaters are not obstructive and can be more aesthetically pleasing than fixed structures. They can easily be rearranged in a different layout and transported to another site for maximum efficiency.

However, in addition to the aforementioned advantages there exist a few disadvantages of surface-piercing breakwaters. Surface-Piercing breakwaters are ineffective in reducing wave heights for slow waves. Surface-Piercing breakwaters are susceptible to structural failure during catastrophic storms. Relative to conventional fixed breakwaters, surface-piercing breakwaters require a high amount of maintenance. The first surface-piercing breakwater appeared in 1811 at Plymouth Port in England. During World War II, Bombardon surface-piercing breakwaters were used along the Normandy coast. In 1930, Japan placed the first surface-piercing breakwater in Aomori Port to test the structure's resistance to waves and the wave dissipation function. At present there are several types of surface-piercing breakwaters in use, which include box, pontoon, mat, tethered float, flexible breakwater etc. Most box type breakwaters are reinforced concrete rectangular shaped modules. These structures have proved to be effective and have a 50 year design life. However, the main disadvantages for these structures are that they are considerably more expensive and require higher maintenance than mat and flexible breakwater type.

In areas where environmental considerations must be evaluated, bottom-standing type breakwaters are considered more frequently as soft solution in solving coastal engineering problems. Bottom-Standing breakwaters provide opportunities for environmental enhancement, aesthetics (one of the major engineering priorities at the moment) and wave protection in coastal areas due to their characteristics that are not found in conventional breakwaters. These characteristics include the ability to promote water circulation and provide a fish habitat enhancement capacity. The bottom-standing breakwaters are being used for fish farming in coastal fishery because they create a calm region in the downstream of the wave motion and act as a sheltered region for a large group of marine habitats during severe wave conditions. These breakwaters resist the sediment transport

and provide a strong protection against coastal erosion. Furthermore, they do not have so many disadvantages as the hard structures like groins, detached breakwaters, revetments, seawalls, etc.

Bottom-Standing breakwaters of different types with their crest at or below the still water level can offer potentially economic solutions in situations where only a partial protection from waves is required. Bottom-Standing breakwaters are generally less expensive to construct and maintain and they offer protection to shorelines and beaches in their natural environment against waves. Further they maintain exchange of water between the protected area and the open water without hindrance apart from maintaining aesthetic appearance without impairment. As the bottom-standing breakwater in general does not extend above the free surface, they cannot be used to stop complete wave motion in the protected areas but it is sufficient for many practical applications. For bottom-standing breakwaters the greater the submergence, the lesser the impact of wave energy on structure, and hence less effective the structure will be for wave attenuation.

1.4 OBJECTIVE OF PRESENT INVESTIGATION

A water wave model that can accurately simulate various aspects of wave transformation in coastal regions is a valuable engineering tool. Such a tool is very useful in handling problems like wave scattering/trapping by onshore and off-shore breakwaters. In the present thesis mathematical models are developed to analyze the water wave scattering and trapping problems by both rigid and flexible breakwaters (with and without porosity) in a two-layer fluid. Suitable computer codes have been written to solve the various physical problems. Major attention is given on the following aspects.

- Generalization of single-layer wave structure interactive model to two-layer systems.
- Investigation of surface- and internal wave scattering.
- Evaluation of efficiency of flexible porous breakwaters in a two-layer fluid.
- Study of surface and internal wave trapping between the breakwater and a vertical rigid end wall.

- Evaluating the influence of fluid density and interface location on the effectiveness of breakwaters in a two-layer fluid.

A class of problems dealing with scattering of harmonic surface- and internal-waves by various breakwaters, namely (i) Rigid dikes, (ii) Porous membrane and (iii) Porous flexible plate in a two-layer fluid are studied in the present work. The trapping phenomenon of surface- and internal-waves by partial porous flexible breakwaters is also investigated. Under the assumption of the linearized theory, these class of physical problems are reduced to a class of two-dimensional mixed boundary value problems associated with Laplace's equation for the determination of the velocity potentials along with the important physical quantities, like, the reflection and transmission coefficients of an incident time-harmonic surface-/internal-wave. A general structural response equation is solved in case of flexible structures like, porous membrane and porous plate, where the response is coupled with the velocity potential. Eigenfunction-expansion, wide-spacing-approximation (WSAM) and least-squares-approximation methods are the main mathematical techniques, which are utilized in the solution of the present mathematical models.

The present Chapter 1 has provided a general introduction, scope and objectives of the present study. In Chapter 2, an elaborate review of literature and the motivation for the present investigation are presented. The basic mathematical tools utilized in the thesis along with the derivations of the basic hydrodynamics and structural response equations in the linearized set up are elaborated in the Chapter 3. In Chapter 4, surface- and internal-waves scattering by a single surface-piercing rigid dike is solved numerically within the context of linearized theory of water waves. After solving this physical problem the study is extended to a pair of identical rectangular surface-piercing dikes. The surface- and internal-waves scattering by a single bottom-standing rectangular rigid dike and a pair of bottom-standing rigid dikes are presented in Chapter 5. Computed results in two-layer fluid are compared with those existing in the literature for a single-layer fluid. The results obtained by the matched-eigenfunction-expansion method are compared with that of WSAM. In Chapter 6 and Chapter 7, surface- and internal-waves scattering by flexible porous structures is considered. The surface- and internal-wave trapping by porous and

flexible partial breakwaters near the end of a semi-infinitely long channel is studied in Chapter 8. In Chapter 9, the summary and the conclusions of the present investigations are presented. The scope of the present research work is also discussed. Bibliography is included separately.

Chapter 2

REVIEW OF LITERATURE

2.1 INTRODUCTION

With developmental activities, the dynamic equilibrium of the coastal region is disturbed, often resulting in coastal erosion and accretion. Coastal erosion is a severe problem worldwide threatening the coastal properties, causing degradation of valuable land and natural resources, disruption to fishing, shipping and tourism. The development of coastal facilities has necessitated proper management of the sea front warranting construction of coastal protective structures. The choice of the structure depends on the wave environment and the morphology of the coastal region. Hence it is very essential to understand the wave-structure interaction problems to predict the behavior of different breakwaters at various wave environments.

Wave-structure interaction is a classical problem in coastal engineering. Such a problem is related to a number of engineering concerns such as structural stability under wave attack, scour in front of structures, reduction of transmitted wave energy, etc. When analyzing the problem of waves interaction with breakwaters, the viscous effects are usually neglected. Therefore, the flow is analyzed using potential flow theory. Linear-wave theory is the core theory and is used extensively in ocean wave modeling over past several years. It is one of the most useful model which has found wide applications over various coastal engineering problems. Moreover, the linear-wave theory acts as an stepping stone

for all non-linear theories, which have been developed over the years to solve various difficult wave mechanics problems. Hence in the coastal engineering applications, linear-wave theory is often used to solve various physical problems and acts as a foundation for the non-linear studies. This chapter is divided into two sections. First an extensive review is carried out describing the earlier works on dynamics of linear surface-waves passing submerged and floating breakwaters (rigid/flexible/porous breakwaters) in a single-layer fluid. The second section deals with the review for previous works on the wave structure interaction in a two-layer fluid within the context of linearized theory of water waves.

2.2 WAVE-STRUCTURE INTERACTION IN A SINGLE-LAYER FLUID

The dynamics of ocean surface-waves is a topic that has been studied extensively. Some of the well known results are contained in the books by Dean and Dalrymple (1991), Mei (1992) and Sorensen (1993). In coastal areas, waves can be partially reflected by rapid changes in the seafloor. This power of reflection due to variation in bottom depth has led to numerous studies and the creation of immersed coastal structures designed to protect the coastal installations or natural shores from wave impact by partial reflection of its energy (see Rey (1992) and Rey (1995)).

On the other hand, the suspended breakwaters have advantages over those attached to the seabed, since they can be used for larger depths and therefore are not limited to small distances from the beach. These type of breakwaters often provide a cost effective and efficient solution of protection from wave attack in semi-protected regions. They may be preferred in deeper waters, in areas where poor foundation conditions or environmental constraints exist or where protection is required only on a temporary basis, such as pollution control, salvage operations, and construction and maintenance of offshore platforms. Such structures could also protect floating airports and portable ports. In this section the review for surface-wave interaction with floating and submerged breakwaters is carried out under three subsections namely rigid, flexible and porous breakwaters.

2.2.1 Rigid Breakwaters

Rigid breakwaters extending to sea-surface have been constructed in the past twenty years to protect pleasure beaches and shorelines against erosion, especially in Japan. Recently, the human desire to protect the beauty of shore landscape and the interest of development of marine recreational areas convinced people to think and apply different and more environment friendly solutions. One solution, which can meet these needs, is submerged breakwaters like dikes instead of detached breakwaters. Because of their deep submergence, ships may pass over them and also the sea-area is available for recreation purpose. Such structures also could be rapidly deployed to protect a harbor or moored vessels from the destructive effects of water waves.

Although rigid submerged breakwaters have proven to be less efficient than emerged breakwaters in reflecting waves, they may be used efficiently as a means of erosion control (Bruno (1993)). The interaction between water and a submerged marine structure has attracted increasing attention over the past decade. Not only are the submerged marine structures related to many ocean and coastal engineering problems, such as dike applications for preventing beach erosion, but also to the water wave propagation over coral reefs or continental shelf. Much of the research attention in recent past has focused on the floating rigid breakwater and there are numerous studies, both experimental and numerical that are reported in the literature. With increasing availability of computational capability, the complex flow configuration near the submerged and floating structures can be analyzed.

Extensive experimental and theoretical investigations have been conducted to examine the performance of different geometries of bottom-standing rigid breakwaters. Since the submerged structures are usually used to reduce the transmitted wave, most of the studies are mainly concerned with determining the reflection and transmission properties for a given incident wave. For standard geometries, less computationally expensive methods are available to solve the wave structure interaction problems. An explicit solution for the scattering of waves by a pair of surface piercing vertical barriers in deep water has been given by Levine and Rodemich (1958). For bottom-standing rectangular

bodies approximate solutions for long waves have been developed by Ogilvie (1960), for long obstacles by Newman (1965), and for low draft structures by Mei (1969). Newman (1965) obtained an approximate solution for surface-waves elevation in the limit of a long submerged obstacle.

Levine (1965) studied the interaction of oblique waves with a completely submerged circular cylinder near the free surface based on the Green's function. Transmission and reflection coefficients were calculated. When the obstacle is in the form of a thick barrier with rectangular cross section present in water of uniform finite depth, the corresponding water wave scattering problems for normal incidence of a wave train have been investigated by Mei and Black (1969). They use the variational formulation to solve the problem. For a single floating cylinder of rectangular cross-section, Black et al. (1971) also used the variational method to solve the radiation problem (where the body oscillates radiating waves into otherwise calm water) and then used the Haskind relation (Newman (1976)) to deduce the forces due to incident waves. Free surface elevations were obtained for a single cylinder and for two cylinders in series. Garrison (1969) investigated the interaction of an infinite shallow draft cylinder oscillating at the free surface with a train of oblique waves using the boundary integral method. Wave scattering by a circular dock has been considered by Garrett (1971).

A number of authors have considered the two-dimensional problems of the radiation and scattering of waves by two parallel circular cylinders in deep water. Wang and Wahab (1971) have extended the multipole method of Ursell (1949) to analyze the heaving of two rigidly connected half-immersed cylinders. Bolton and Ursell (1973) used the multipole expansion method to the interaction of an infinitely long circular cylinder with oblique waves. The added mass, damping coefficients and vertical wave force were calculated. Bai (1975) presented a finite element method to study the diffraction of oblique waves by an infinite cylinder in water of infinite depth. Reflection and transmission coefficients and the diffraction forces and moments were computed for oblique waves incident upon a rectangular cylinder. Raman et al. (1977) reported on the damping action of rectangular and rigid vertical submerged barriers and expressed the transmission coefficients in terms

of the transmitted energy to the total power of the incident wave. Further it is concluded that in case of rectangular submerged barrier, the top width of the barrier plays an important role in controlling transmission coefficient.

An integral equation formulation for the calculation of hydrodynamic coefficients for long, horizontal cylinders of arbitrary section has been presented by Naftgzer and Chakrabarti (1979). Due to the general complexity of multi-body problems a number of authors have used a WSAM where only interactions are assumed to arise from plane waves traveling between the bodies. The accuracy of this type of approximation has been demonstrated in a number of cases. Two-dimensional problems considered include two surface piercing barriers, by Srokosz and Evans (1979) and two half-immersed circular cylinders by Martin (1984). Martin (1984) has pointed out that the basic assumptions behind the WSAM are that both the wavelength and a typical body dimension must be much less than the separation between bodies. It is a remarkable fact that the WSAM has consistently given good results even when these assumptions are clearly violated.

The test on wave transmission and reflection characteristics of laboratory breakwater conducted by Seeling (1980) indicate that both the depth of submergence and top width are important in determining the performance of the breakwater and it is suggested that for near zero submergence, submerged breakwater is efficient in reducing the transmission. The absorption of wave energy with a submerged cylindrical duct is studied by Thomas (1981). Liu and Abbaspour (1982) studied the scattering of oblique waves by an infinite cylinder of arbitrary shape using a hybrid integral equation formulation and numerical results were presented. The scattering problem for bodies of arbitrary shape may be solved by integral equation methods for which an extensive review is given by Mei (1983). Leonard et al. (1983) extended Bai's (1975) finite element method to solve the diffraction and radiation boundary value problems arising from multiple two-dimensional horizontal cylinders interacting with obliquely incident linear-waves. Hydrodynamic parameters and responses of two cylinders in heave and sway were calculated.

Martin (1984) has solved the scattering problem for two rigidly connected half-immersed cylinders by the null-field method. Garrison (1984) used a Green's function method to

compute the oblique wave interaction with a cylinder of arbitrary section on the free surface in water of finite depth. The equivalent problem of Levine and Rodemich (1958) has been solved in finite depth water by Falnes and McIver (1984) using the method of matched-eigenfunction-expansions. For barriers of differing length the matching is carried out at the two boundaries of three adjoining regions. Similar to Naftgzer and Chakrabarti (1979) an integral equation formulation for the calculation of hydrodynamic coefficients for long, horizontal cylinders of arbitrary section has been presented by Andersen and Wuzhou (1985).

McIver (1986) solved the scattering problem for adjacent floating bridges by a direct method. A matched-eigenfunction-expansion method and WSAM is used for the solution of a number of axisymmetric three-dimensional problems and results from both the methods are compared. For the case of obstacle in the form of a thick wall with a submerged narrow gap in finite depth water, Liu and Wu (1987) used the method of matched asymptotic expansion to obtain an approximate analytical expression for the transmission coefficient assuming the width of the wall to be of the same magnitude as the wavelength. Isaacson and Nwogu (1987) developed a generalized numerical procedure based on Green's theorem to compute the exciting forces and hydrodynamic coefficients due to the interaction of oblique waves with an infinitely long, semi-immersed floating cylinder of arbitrary shape. Numerical results for wave loads and motions were presented. Williams and Darwiche (1988) analyzed the three-dimensional scattering of waves by elliptical breakwaters using eigenfunction-expansions. Their numerical results are valid for the entire wavelength spectrum and finite obstacle length.

Subcritical and supercritical solutions for surface-waves over circular bumps were given by Shen et al. (1989). Chakrabarti and Naftgzer (1989) evaluated the wave forces on a submerged semi-cylinder resting on the bottom using a boundary integral method. The non-dimensional horizontal and vertical forces were obtained at different values of the wave number. Kobayashi and Wurjanto (1989) presented a numerical approach based on finite amplitude shallow water equation for determination of wave transmission over submerged breakwater. They obtained the approximate solution in the limit of long

waves. Design equations for wave transmission over a submerged breakwater are detailed by Rojanakamthorn et al. (1989) based on mild slope equation and the result suggests that when the height of the submerged breakwater is 50% of the total depth, the transmission coefficient varies between 0.4 and 0.7. For the normal incidence of waves most of the problems can be solved explicitly.

Rey et al. (1992) conducted wave tank experiments over a rectangular submerged bar. Beji and Battjes (1993) also conducted laboratory experiments to study the generation of higher harmonics by a submerged trapezoidal bar. Mullarkey et al. (1992) utilized an eigenfunction-expansion approach to calculate the hydrodynamic coefficients for rectangular TLP pontoons. Drimer et al. (1992) presented a simplified approach for a floating breakwater where the breakwater width and incident wavelength are taken to be much larger than the gap between breakwater and the seabed. Losada et al. (1992) and (1993) applied the eigenfunction-expansion method to the propagation of oblique waves past rigid vertical thin barriers and calculated the transmission and reflection coefficients. Sannasiraj and Sundaravadivelu (1995) applied the finite element technique to study the interaction of oblique waves with freely floating long structures. The hydrodynamic behavior of two-dimensional horizontal floating structures under the action of the multi-directional waves has been investigated, and the motions and forces on a rectangular floating structure experiencing unidirectional and multi-directional wave fields were computed.

Abul-Azm (1994a) investigated the diffraction through wide submerged breakwaters under oblique waves by use of the eigenfunction-expansion method and discussed the effect of different wave and structural parameters on the transmitted and reflected waves and the hydrodynamic loadings on the breakwater. Mandal and Dolai (1994) used the one-term Galerkin approximation to determine the upper and lower bounds for the reflection and transmission coefficients in the problems of oblique water wave diffraction by a thin vertical barrier in water of uniform finite depth. Mallayachari and Sundar (1996) investigated numerically the wave reflection characteristics over submerged rectangular step of finite length. They compared reflection characteristics of rectangular structures

with half cylinder and trapezoidal obstacles. Ertekin and Becker (1996) applied a finite difference method to examine diffraction of waves by a submerged bottom-mounted trapezoidal obstacle. For oblique incidence of the wave trains and/or for finite depth water, the problems cannot be solved explicitly and they can be tackled by some approximate methods to obtain numerical estimates for some physical quantities such as the reflection and transmission coefficients (Sudeshna et al. (1996)).

Isaacson et al. (1996) experimentally investigated the reflection of obliquely incident waves from a model of rubble mound breakwater. Results show that both the reflection coefficient and the reflected phase lag are noticeably dependent on the angle of incidence and that the variation with the angle of incidence further depends on the depth to wavelength ratio. Abul-Azm and Williams (1997) used the eigenfunction-expansion method to examine oblique wave diffraction by a detached breakwater system consisting of an infinite row of regular-spaced thin, impermeable structures located in water of uniform depth. A comprehensive review of wave reflection by uneven bottom has been presented in the book of Dingemans (1997). The one-term Galerkin approximation was used also by Das et al. (1997) to evaluate the upper and lower bounds for the reflection and transmission coefficients in the problem of oblique water wave diffraction by two equal thin, parallel, fixed vertical barriers with gaps presented in water of uniform finite depth. Sannasiraj et al. (2000) used the finite element method to the study of the diffraction-radiation of multiple floating structures in directional waves.

Abul-Azm and Gesraha (2000) examined the hydrodynamic properties of a long rigid floating pontoon interacting with linear-waves in water of finite depth by use of the eigenfunction-expansion method. This kind of structure was known as a kind of effective breakwater and many investigators (e.g., Drimer et al. (1992), Cheong et al. (1996), Williams et al. (2000) and Zheng et al. (2004)) have studied the radiation and/or diffraction problem under the action of normal incident waves. Li et al. (2002) explored the interaction of oblique irregular waves with a vertical wall. The ratio of the oblique wave forces to the normal incident wave forces was given and the characteristics of the reflection coefficients for oblique waves were introduced.

Politis et al. (2002) developed a Boundary Integral Equation (BIE) method for oblique water wave scattering by cylinders in water of infinite depth and four geometrical configurations were chosen to investigate the numerical performance of the BIE method. The added mass, damping coefficients and excitation forces were calculated. Recently Söylemez and Gören (2003) studied the diffraction of oblique water waves by thick rectangular barrier mounted on seabed. They also studied the diffraction of oblique water waves by thick rectangular barrier floating at the free surface experimentally and investigated theoretically.

The first ever analysis on the problem of trapping of surface-waves over a uniform sloping beach was carried out by Stokes (1846). This phenomenon of wave trapping analyzed by Stokes is referred as edge waves which can travel unchanged in the direction of the shoreline, and decays exponentially to zero in the seaward direction. In spite of the unbounded fluid region, this Stokes edge wave is confined or trapped by the sloping boundary. The understanding of trapped waves in various physical situations is important in various coastal engineering applications like dynamics and sedimentology of the near shorezone through their interaction with ocean swells and surfs (Leblond and Mysak (1978)).

There has been a significant interest in the literature to understand the existence of trapping waves and many researchers found out the wave frequency corresponding to the trapped mode in different physical situations within the context of linearized theory of water waves. Ursell (1951) proved the existence of trapped waves above a submerged horizontal cylinder of sufficiently small radius in a channel spanning the sidewalls. Jones (1953) generalized Ursell's result to submerged cylinders of arbitrary but symmetric cross section in finite water depth. Wave trapping may occur along a vertical wall when there is an abrupt change in ocean depth (Leblond and Mysak (1978)). The existence of trapped waves above a submerged horizontal plate was investigated by Linton and Evans (1991) based on matched-eigenfunction-expansion method. Evans and Kuznetsov (1997) gave a detailed review of the development in the recent decades that has taken place on the existence of trapped waves. In all these study, emphasis is given on the wave frequency

i.e., on the trapped mode.

2.2.2 Flexible Breakwaters

Rigid structures like submerged dikes have major disadvantages like requirement of vast cross-sectional areas and therefore demanding high construction costs. Furthermore, under most considerations, marine bodies are assumed rigid in the presence of waves and their elastic deformations are neglected. However, the hydroelastic effect should be considered under certain wave conditions, as when (i) the body itself is flexible, (ii) the body is very thin compared to wave parameters, and (iii) the body is very long with respect to the incident wavelength. The former two cases should be quite obvious. However, in the latter case, localized deflection or vibration of a long structure becomes significant due to the continuous excitation of small amplitude waves, although the motion of the whole body is small as compared to its length.

The interaction of gravitational waves with deformable structures like plate, membrane and ice is of interest in the study of the dynamics of off-shore/on-shore structures, breakwaters, ice fields and artificial structures (airports and islands) when acted upon by sea waves. Floating and/or flexible wave barriers have generated interest amongst researchers with some of its added advantages. They can be moored despite unfavorable seabed soil conditions and are usually positioned near the water surface where the wave energy is most pronounced. They allow for free passage of seawater, fish and sediment transport beneath, thus being friendlier to the environment. They can also be more economical compared to fixed-type breakwater especially when the water depth is large, and can be prefabricated onshore thus reducing construction time.

A great deal of effort is spent to effectively utilize the ocean space for human activities and developments. Certain huge platforms, which are in general flexible are constructed or extended from shoreline to provide more dry space, while structures like floating ports, mobile offshore bases are built as working spaces. In recent times, several artificial floating islands are constructed off shoreline. Such huge floating structures are categorized as Very Large Floating Structures (VLFS). Before the construction and positioning of any VLFS,

careful and detailed studies are needed to investigate the hydrodynamic performance and hydroelastic behavior of not only the VLFS system but also the system of breakwaters protecting the VLFS.

In the theoretical study for the modeling of wave interactions with flexible structures, there are two major approaches. These are the mode expansions method and the eigenfunction-expansions method. In the mode expansions method, the body deformation is represented by a series of natural modes. The kinematic and dynamic surface conditions due to elasticity and gravity are treated separately. Based on the mode expansions method, Bishop and Price (1979) gave a comprehensive summary on the studies of hydroelasticity of ships, and Gran (1992) summed up the engineering applications of structural responses of marine structures to waves. Kashiwagi (2000) gave a review of the developments on VLFS and reported that major works on VLFS are based on the mode expansions method. However, the mode expansions method is only applicable to a finite plate. On the other hand, the eigenfunction-expansions method is a more direct method, as it combines the kinematic and dynamic surface conditions, which give the dispersion relation satisfied by the wave numbers.

Fox and Squire (1994) used the eigenfunction-expansions method to study the interaction of surface-waves with an ice-covered surface and obtained the solution by the conjugate gradient method. They observed that the eigenfunctions are not orthogonal with respect to the conventional inner product, though the eigenfunctions are complete. Squire et al. (1995) presented an invited review on the interaction of gravity waves with an ice-covered surface. Balmforth and Craster (1999) developed a method based on the Fourier transform and Wiener-Hopf technique to study the scattering of gravity waves incident on an ice-covered ocean and obtained asymptotic and approximate solutions to the problem. They considered ice as a thin elastic plate in the mathematical model. To analyze the response of a thin horizontal elastic plate floating in waves, Ohkusu and Nanba (1996) combined the kinematic and dynamic surface conditions to obtain the free surface condition on the plate-covered surface and then solve the problem by the boundary integral method.

Sturova (1998) used Fox and Squire's (1994) approach to study the oblique incidence of surface-waves onto an elastic band. Kim and Ertekin (1998) constructed a complete set of orthogonal eigenfunctions satisfying the dispersion relations and then obtained the solution explicitly for predicting the hydroelastic behavior of a shallow-draft VLFS. Nanba and Ohkusu (1999) analyzed the elastic response in waves of a large floating platform of thin plate configuration in both shallow water and deep water. The free surface condition gives an important information regarding the wave numbers. In all the aforementioned studies related to an elastic plate floating on the water surface, the plate is assumed to have a free edge, which suggests that the shear force and the bending moment of the plate vanish at the edge. However, artificial structures are usually kept fixed or moored at the edge by ropes, anchors, tension cables, or piles. In such cases, the free edge condition or the built-in edge condition as per the reality. It may be noted that for the simply supported edge condition, the deflection and the bending moment are assumed to vanish, whereas for the built-in edge condition, the deflection and the slope of deflection will vanish.

In the mode expansion method, Newman (1994) proposed to employ different orthogonal polynomials to represent the corresponding mode expansions for different edge conditions, and claimed that the application of natural modal functions should be limited to the free edge condition only. He noted that it is very difficult to identify the fittest modal functions for various edge conditions. Wu et al. (1995) extended Newman's (1994) idea to analyze the wave-induced responses of an elastic floating plate. Sahoo et al. (2001) investigated the interaction of surface-waves with a semi-infinite elastic plate floating on the free surface in finite water depth. The hydrodynamic behavior due to three different types of edge conditions, namely (i) free edge, (ii) simply supported edge, and (iii) built-in edge is analyzed. They used a newly defined inner product along with the method of matched-eigenfunction-expansions, to obtain the full solution. The edge conditions are directly incorporated while using the matching condition along with the orthogonality property. The inner product defined in their work, is a generalization of the well-known gravity wave inner product developed by Havelock (1929), which was gener-

alized by Rhodes-Robinson (1979a) and Rhodes-Robinson (1979b) to deal with problems related to capillary gravity waves.

Submerged flexible breakwaters have received increasing attention recently in the protection of pleasure beaches and shoreline from erosion. A considerable amount of research has been devoted to investigate various problems dealing with submerged flexible breakwaters. For coastal zones where water depth increases rapidly the use of concrete caissons or rubble mound structures as wave-barriers are expensive. In view of this, a horizontal plate, submerged a finite depth beneath the sea surface and supported on a group of piles, that requires less concrete per unit run, is suggested as a possible type of breakwater, in relatively large water depths (Neelamani and Reddy (1992)). A horizontal single surface plate fixed at the free surface (Stoker (1957)), a submerged horizontal plate (Siew and Hurley (1977), Patarapanich (1978), and Wang and Shen (1999)), a group of submerged plates (Wang and Shen (1999)) and a horizontal double-plate breakwater consisting of a seaward submerged plate and a leeward surface plate (Cheong and Patarapanich (1992)) are some of the breakwaters used and several theoretical and experimental reports are available on the hydrodynamic performance of these breakwaters and shore protection structures.

Number of investigations on different hydrodynamic aspects of horizontal breakwaters are reported in literature. Since the pioneering study by Heins (1950) on the wave diffraction from a semi-infinite submerged horizontal plate in finite depth and the investigation by Stoker (1957) on the reflection and transmission coefficient of long waves propagating past a surface plate in shallow water, a great deal of effort is spent on obtaining optimum conditions for minimum wave transmission and obtaining design data of such structures (Burke (1964), Dick (1968), Hattori and Matsumoto (1977), Siew and Hurley (1977), Patarapanich (1978), and Wang and Shen (1999)). Evans and Morris (1972) treated the problem of reflection and transmission of oblique waves by a vertical fixed plate-type barrier in deep water. The method of matched asymptotic expansions has been employed by Siew and Hurley (1977) and analytical expressions for the reflection and transmission coefficients for long waves propagating past a submerged plate in shallow water have been

obtained.

Patarapanich (1978) has examined the averaged energy flux across various regions around the submerged plate (with restriction to shallow water case) and has analyzed the variation of the reflection coefficient with plate length and has derived conditions of maximum and zero reflection. Wave propagation over a submerged plate has also been modeled by general numerical methods, such as the boundary integral method developed by Liu and Iskandarani (1989) and the finite element method by Patarapanich and Cheong (1989). Experimental results conducted by Patarapanich and Cheong (1989) show that the optimum plate width is about 0.5-0.7 times that of the wave length above the plate, for the plate submergence of around 0.05-0.15 times the water depth, for minimum transmission of waves. Liu and Iskandarani (1991) and Yu et al. (1991a,b) have provided various information related to possible engineering applications. Neelamani and Reddy (1992) have experimentally investigated the hydrodynamic performance of a submerged horizontal plate, for different depths of submergence of the plate and for a wide range of wave steepness in deep water conditions. Yip and Chwang (1996) have presented an analytical solution to describe a submerged pitching plate as an active wave controller by eigenfunction-expansion method.

The above investigations have considered the wave motion over a single submerged plate. However, there are many occasions, where a submerged plate may not provide the required wave protection due to the flow in the relatively longer region between the plate and the seabed. In view of this, the reflection and transmission coefficients of a horizontal double-plate system consisting of a seaward surface plate and a submerged leeward plate have been derived in terms of reflection and transmission coefficients of a single plate case by Cheong and Patarapanich (1992). Their experimental measurements show that the optimum degree of submergence of the leeward plate should be about 0.1-0.2 for minimum wave transmission. The reflection coefficient for the double-plate system has been observed to be greater than that for a single plate system, for all submerged depth ratios. Their results suggest a large fraction of reflected wave energy in the double-plate system. Further, the experimental results show that the transmission coefficient is

a minimum at relative depths of about 0.15-0.20. This range also corresponds to higher loss coefficients, which tend to remain relatively high even at larger depths. This suggests that the double-plate combination is more effective than the single plate case at larger relative depths.

Wang and Shen (1999) have considered wave motion over a group of submerged horizontal plates. The method of eigenfunction-expansions has been applied to obtain the velocity potentials and the free surface elevations. The unknown constant coefficients have been determined from the matching conditions, using three sets of orthogonal eigenfunctions. The success of double-plate breakwater developed by Cheong and Patarapanich (1992) in serving as an effective wave-barrier has motivated Usha and Gayathri (2005) to consider the case of wave motion over a twin-plate system consisting of a surface plate on the free surface and a submerged plate of same dimensions just below the surface plate, for different submergence spacing of the lower plate. The main objective of the study was to evaluate the hydrodynamic performance of the twin-plate system as a breakwater and compare its performance with those of other available horizontal breakwaters. The study was restricted to two-dimensional cases of waves approaching normally to the breakwaters. The method of eigenfunction-expansions was used to obtain the velocity potentials and the unknown constant coefficients are determined from the matching conditions, using three sets of orthogonal eigenfunctions. The eigenfunction-expansion method combined the kinematic and dynamic surface conditions which give the dispersion relation satisfied by the wave numbers. This method has been successfully and effectively employed by several investigators including Wang and Shen (1999), Sahoo et al. (2001), Fox and Squire (1994) and Hu et al. (2002) in their analysis of wave motion over submerged structures or water transmission through a porous medium or on the hydrodynamic performance and the hydroelastic behavior of the system.

There is a considerable interest among researchers over past several years to analyze the effectiveness of vertical barriers in attenuating wave energy. A huge system of flap-type barriers has been planned in Italy to protect the lagoon of Venice against high tides (Natale and Savi (1993)). Bai (1975) studied the reflection and transmission coefficients

and the diffraction forces and moments for oblique waves incident upon a vertical flat plate. Studies using linear theory for their reflection power as a function of wavelength have shown the presence of maxima and minima in the reflection (Patarapanich (1984) and Sturova (1991)).

The solution of the flexible emerged breakwater problem has been treated dynamically as a one-degree-of-freedom system (see, Leach et al. (1985) for a single hinged flap and Sollitt et al. (1986) for double flaps). Evans and Linton (1990) suggested that submerged floating breakwaters may be tuned to the incoming waves to provide more reflection than emerged, surface-piercing structures. They had treated their system as a single-degree-of-freedom system. The effect of the breakwater rigidity had been considered by Lee and Chen (1990) for a hinged flap breakwater. Effects of flexibility and buoyancy had been introduced by Williams et al. (1991) for one compliant beam-like clamped breakwater using an appropriate Green function, and by Abul-Azam (1994) for a double fixed or hinged flap using an eigenfunction approach.

Williams et al. (1992) had extended the numerical solution of Williams et al. (1991) to a submerged compliant and clamped breakwater problem. They used an integral equation method, which includes a laborious technique for finding an appropriate Green's function. This method is very much useful and computationally attractive when more complicated geometries are considered (Natale and Sivi (1993)). Williams (1993) analyzed the wave diffraction due to a pair of flexible breakwaters consisting of compliant, beam-like structures, also anchored to the sea bottom and kept under tension by a small buoyancy chamber at the top. Abul-Azm (1995) presented an analytical solution for the submerged breakwaters, based on expressing the linear hydrodynamic wave potential in terms of eigenseries expansion. This expansion technique had been shown to be a computationally efficient method for several linear and non-linear problems (see, Abul-Azm (1993)). The breakwater is considered to be flexible beam-like, either clamped or hinged at the seabed, anchored to the seabed by tethers and kept under tension by means of a floating buoy at the breakwater tip. The assumptions are the same as used by Williams et al. (1992).

Recently, flexible barriers consisting of vertically tensioned membranes spanning the

entire water depth were reported. The flexible membrane, which is light, inexpensive, reusable, and rapidly deployable, can be used as a portable and sacrificial breakwater. Since it can be easily removed, it is considered as having minimum environmental impacts on various coastal processes. They can be easily fabricated in large sizes, allowing for wave control in a wide region. The multi-mode motions of the membrane can also be explored to widen the effective frequency range for wave attenuation, especially on irregular waves. Kim and Kee (1996) and Kee and Kim (1997) reported results for a single membrane with the latter incorporating the effects of a buoy. Both the eigenfunction and the boundary integral method were used in their studies.

Using the eigenfunction approach, Lo (1998) and Abul-Azm (1994b) investigated the cases of the dual flexible membrane and dual hinged beams respectively, while Cho et al. (1998) used the boundary integral method for dual membranes tensioned by buoys. Lo (2000) further considered a flexible membrane of finite length with gaps between the membrane and the water surface and/or the bottom. In the studies for a membrane system without buoys, a typically fixed boundary condition is applied at the upper edge of the vertical membrane that coincides with the water surface. In practice, the tension on the membrane will have to be applied through buoys or through cables above the water surface. The effect of tides also implies that a membrane will have to protrude above the mean water surface in order to intercept the waves at high tide conditions. The scattering effect of buoys have been examined numerically and experimentally by Kee and Kim (1997) and Cho et al. (1998) in their studies using the boundary integral method. Cho et al. (1997) studied the performance of flexible membrane wave barriers in oblique incident waves. The performance of surface-piercing or submerged buoy/membrane wave barriers is tested with various membrane, buoy, and mooring characteristics and wave conditions including oblique wave headings.

Chan and Lee (2001) investigated wave characteristics past a flexible fishnet. Lee and Lo (2002) reported the effect of having the membranes protrude above the water surface such as that rising from having tension provided by frame with surface cables. They considered an array of vertical membranes of arbitrary draft and protrusion. The

eigenfunction-expansion method is used assuming linear-wave theory and small membrane motion. A mixed dynamic boundary condition results at the plane of the membrane and is solved by the method of minimal squares. Computational as well as experimental results are presented for single and dual membrane arrangements. Viscous energy loss is also incorporated for better comparison of the experimental data with numerical prediction.

In the recent past there have been attempts to solve and understand the complex physical problems such as analyzing the wave trapping by means of flexible structures. Using the method of hypersingular integral equations Parsons and Martin (1995) investigated the trapping of water waves by submerged plates in water of infinite depth and discussed the results of inclined flat plates and for curved plates that are symmetric with respect to a line drawn vertically through their centers.

2.2.3 Porous Breakwaters

Recently porous breakwaters are reported to be a preferred choice as wave attenuating breakwaters for various coastal engineering applications in the literature. There are widespread applications of porous structures in coastal engineering. A porous structure allows the partial transmission of water waves with energy dissipation. Meanwhile, the wave heights in front of the porous structure and the wave-induced forces on the structure can also be reduced. However, the porosity of the breakwater presents a challenge to any attempt at a predictive model. This is because porosity can change the complexity of the flow in a number of ways during various wave conditions.

The theoretical study of wave motion through a porous medium depends on establishing a general theory of porous medium flow. However, even in its simplest form, this physical phenomenon is extremely complicated. A common theme of research on the surface-waves and porous medium interaction during the past decade has been on the understanding of (a) wave motion past a porous medium, (b) a porous medium as an active wave controller and (c) stability of a porous medium under wave attack. In fact, this subject has found applications not only in the harbor but, more importantly, in the laboratory, where testing and experiments of designs are carried out before they are put

into actual use.

In past there were many attempts to understand the water wave interaction with various porous structure configurations. It is found that using variable permeability allows the determination of results for solid obstacles with any number of holes. One classic example of such problem is that considered by Ursell (1947), who investigated waves past a surface-piercing barrier reaching partway to the bottom in infinitely deep water. Considerable effort has been devoted to achieving a good understanding of the phenomena of harbor resonance. Jarlan (1961) was the first to study the use of a perforated vertical wall breakwater to reduce wave energy. Significantly progress in the study of harbor resonance was made by Miles and Munk (1961) and Le Méhauté (1961). Lewin (1963) studied the reflection of waves by a barrier with any number of gaps in infinitely deep water. His solutions were complicated and no numerical results or concrete examples were presented. The studies conducted by Dick and Brebner (1968) on solid and permeable type submerged breakwaters indicate that infinitely long porous structures with near zero submergence are capable of reducing the incident wave energy by 50%. In finite depth, the problem of Ursell (1947) has been studied numerically by Mei and Black (1969).

Rigorous solutions to harbor resonance problem were also presented by Lee (1971), who considered rectangular and circular harbors with opening located on a straight coastline. Although many important aspects of the resonance phenomenon can be understood by considering a geometrically idealized harbor, any practical application depends on the establishment of numerical theories or codes that are able to deal with arbitrary harbor configurations (Chwang and Chan (1998)). Such theories are available after Lee (1971). Tuck (1971) and Porter (1972) derived formal solutions for the transmission of water waves through a thin plate with a small gap in infinite water depth using potential theory. Guiney et al. (1972) extended Tuck's (1971) theory to incorporate a finite barrier thickness and obtained similar results, both theoretically and experimentally. They found that at very low frequency the thickness of the barrier has little effect. However, at and above the frequency of maximum energy transmission, the point where the transmission is maximum, the effect of thickness on reducing energy transmission is pronounced. The

frequency at which maximum transmission occurs also tends to be slightly lower for thick barriers.

Le Méhauté (1972) also studied a wave motion past vertical plates placed parallel to the direction of flow to act as a wave absorber in a wave flume. These plates cause partial transmission and reflection of the incident wave because of the sudden variation of permeability as well as because of the increase of laminar boundary layer. The idea is actually similar to a thick barrier with multiple slits. In most of existing studies it is assumed that the porous medium obeys Darcy's law. For wave motion across a porous medium, Sollit and Cross (1972) gave a modification to Darcy's law that was later used extensively by many workers in the field. In their approach, the force exerted by the porous medium on the flow was assumed to be composed of two components; (a) a resistance force, linearized through an interaction procedure (described below) in accordance with Lorentz's principle of equal work, and (b) an inertial force, which is linearly proportional to the fluid acceleration and is represented by an added mass coefficient. The concept of Sollit and Cross (1972) is also used in the famous Morison equation by Sarpkaya and Isaacson (1981) to predict wave forces on small rigid bodies. Raichlen (1974) reviewed some of the experiments on the effect of waves on rubble-mound structures and provided much experimental evidence and information about the characteristics of porous structures as breakwaters.

Madsen (1974) worked out the wave transmission and reflection characteristics of a long wave train past a single porous medium, assuming the value of inertial coefficient as one. The theoretical work of Madsen (1974) also show reasonable agreement with those of other theories and experimental works like those of Kondo and Toma (1972), Nasser and McCorquodale (1975), and Qui and Wang (1996). Analyzing the same problem as that of Le Méhauté (1972) using a slightly different approach, Tuck (1975) obtained equivalent results. Mei and Chen (1975) developed the hybrid-boundary-element technique for harbor engineering. Macaskill (1979) made a comprehensive analysis and numerical study of the wave motion past a thin permeable barrier. His method involves the use of Green's function and can be generalized to study the flow past a barrier with any number

of slits at different locations, which is convenient for computational analysis. Macaskill's (1979) and Tuck's (1975) wave characteristics results in the presence of a porous plate show some similarity, although not exactly equivalent trend. Macaskill (1979) derived an empirical formula to relate the geometric permeability to that of Darcy's permeability. However, essential differences between the definition of permeability or porosity exist between Macaskill's (1979) analysis and Darcy's law, and this makes reconciliation difficult. Although this first approximation can be used in an attempt to compare the two results, the formulae are only valid for a certain range of permeability and cannot be regarded as in general agreement (Chwang and Chan (1998)). When a porous medium is itself subject to oscillation, it can also behave as an active generator for wave motion. The study of this subject was initiated by Madsen (1970). He analyzed the influence of leakage around a piston-type wavemaker and found that the leakage effect was great in reducing the wave amplitude. The porous effect on the wavemaker can find applications in the study of surface-waves in reservoirs or lakes caused by landslides during earthquakes.

Another possible application of a porous wavemaking mechanism is in situations where the efficiency of the generation of waves is of interest. A porous wavemaker may be helpful in reducing the total load that is accompanied by a reduction of wave amplitude. Chwang (1983) produced a comprehensive analysis of a thin porous wavemaker. Chwang's (1983) theory was extended by Chwang and Li (1983) to analyze the waves generated by a porous wavemaker near the end of a semi-infinitely long channel of constant depth. Chakrabarti (1989) used an integral transform method to extend Chwang's (1983) theory when the effect of surface tension is also taken into consideration. His results show that surface tension reduces the wave amplitude slightly but does not greatly affect the wave characteristics. Chwang and Dong (1985) studied analytically the wave motion past a thin vertical porous plate in an infinite or semi-infinite wave flume. They assumed that the porous plate is thin hence following Taylor (1956), who proposed that the seepage velocity can be assumed to be linearly proportional to the pressure difference across the two sides of the plate.

Evans (1990) studied theoretically the use of multiple porous screen as a means to

damp the waves in a narrow wave tank. He employed Tuck's (1975) empirical modification to include both inertial and viscous effects. In their proposed empirical equations, the proportionality constant between the pressure jump and normal velocity is analogous to the acoustic impedance in studies of acoustics phenomena as mentioned by Morse and Ingard (1968). Evans's (1990) results for a single screen are in general identical to those obtained by Chwang and Dong (1985) for a single plate. He found that the distance for optimal wave absorption also depends on the wave number and porous coefficient. Dalrymple et al. (1991) studied the reflection and transmission of a wave train at an oblique angle of incidence by an infinitely long porous structure with a finite thickness.

Huang (1991) extended Chwang's (1983) theory to accommodate finite thickness as well as the inertial of a porous plate. Based on Biot's theory of poroelasticity, Huang and Chao (1992) studied the inertial effect of the porous structure. They located a surface of seepage at the interface between the porous structure and the fluid region; that is, the wave profiles at the two regions are discontinuous. This discontinuity causes the velocity to change abruptly at the interface. This velocity singularity has also been noted by many previous workers (Muskat (1946), Richey and Sollitt (1970) and Madsen (1983)) but is usually neglected because of the difficulty of analysis and because of its limited region of influence. Hybrid-boundary-element technique was used by Su (1993) to study harbors of arbitrary geometry. Besides direct applications to the harbor engineering field, the theories of flow past porous medium also have important connections to many surface-wave laboratory experiments. Wang and Ren (1993) presented a theoretical study on the scattering of small amplitude waves by a flexible, porous and thin beam like breakwater held fixed in the seabed. Huang et al. (1993) corrected the earlier solution and also performed a boundary integral approach to verify the validity of a thin porous wavemaker.

Yu and Chwang (1994a) investigated the resonance in a harbor with porous breakwaters with the wave entering at an arbitrary angle. They derived a new boundary condition for the porous breakwaters, which includes the complex porous effect parameter. This modified parameter include both inertial and resistance effects on the porous medium. If the resistance effect in the porous medium is more prominent, the complex porous effect

parameter reduces to the Chwang parameter (Chwang (1983)). They presented a detailed study on the phenomena of wave induced oscillation in a harbor with porous breakwaters. They concluded that the inertial effect of the porous structure is mainly to increase the resonant wave number and it does not reduce the amplitude of the resonant oscillation effectively, while the porous resistance effect reduces the amplitude of the resonant oscillation effectively and gives a little change of the resonant wave number.

Yu and Chwang (1994b) also investigated the wave behavior within the porous medium. They found that as long as the medium impedance is purely imaginary, the porous medium is nondissipative. This confirms the fact that there is no decay of progressive waves in a nondissipative medium. They also note that increasing the absolute value of the reactance of a nondissipative medium has the same effect as increasing the wave frequency or water depth. For slightly dissipative medium, the reactance dominates the resistance. Yu and Chwang (1994b) found that the first-order modification to the progressive wave is a stationary decaying wave with the same feature as evanescent waves in water with no porous medium. This wave causes the leading-order progressive wave to decay. The first-order modifications to the evanescent waves, on the other hand, are progressive waves that emit the leading-order stationary evanescent waves. An interesting phenomenon occurs for a strongly dissipative medium. Yu and Chwang (1994b) found that increasing the medium resistance causes shallow-water waves to be shortened and deep-water waves to be elongated. Besides the direct impact of waves on a porous medium, a considerable amount of work has also been devoted to the diffraction of water waves by a submerged horizontal porous plate. The motive behind this is the possibility of using the submerged porous plate as an alternative to the breakwater, since a submerged breakwater does not visually partition the sea and allows the free exchange of water or marine animals for the benefit of the environment. Factors effecting the reflection, transmission, energy loss and force on the plate include the dimensionless length of the plate, the relative water depth, the submergence depth of the plate and the porosity of the plate.

Yu and Chwang (1994c) employed the boundary integral method to study wave diffraction by a horizontal porous plate submerged at a certain distance below the free surface

in a fluid of constant depth. Chwang and Wu (1994) extended the study to consider wave diffraction by a porous disk, although the essential features of the wave characteristics remain similar. It is observed that the reflection coefficient varies periodically as the length increases. This is a common feature of wave propagation over the submerged solid obstacles (Mei and Black (1969)) and is largely due to the change of celerity when waves propagate in different water depth. Recently, there has been a great deal of effort directed towards quantifying wave interaction with porous ocean structures.

Wave diffraction by a semi-porous cylindrical breakwater protecting an impermeable circular cylinder was investigated theoretically by Darwiche et al. (1994) by an eigenfunction-expansion approach. The interaction of linear water wave in a channel of constant depth, impinging on a vertical thin porous breakwater with a semi-submerged and fixed rectangular obstacle in front, is investigated by Yang et al. (1997). Williams and Li (1998) extended this analysis to deal with the case where the interior cylinder is mounted on a large cylindrical storage tank. Using a least-squares method, Lee and Chwang (2000a) studied the scattering and generation of water waves by vertical permeable barriers. Eigenfunction-expansion method was used by Twu et al. (2002) to examine the wave damping characteristics of vertically stratified porous structures under oblique wave action. It was found that the wave damping efficiency of a vertically stratified porous structure behaves very similar to a simple structure for common angle of incidences and the reflection coefficient decreases with increasing angle of incidence while the transmission coefficient only slightly increases as the angle of incidence increases.

Till date there are only a few studies on wave trapping by porous structure. Sahoo et al. (2000) studied the trapping and generation of surface-waves by submerged vertical permeable barriers or plates kept at one end of a semi-infinitely long channel of finite depth for different barrier and plate configurations. Recently Yip et al. (2002) investigated the trapping of surface-waves by submerged vertical porous and flexible barrier near the end of a semi-infinitely long channel of finite depth. In these problems, emphasis is given on the wave characteristics in the trapped region i.e., between the channel end-wall and the barrier.

2.3 WAVE-STRUCTURE INTERACTION IN A TWO-LAYER FLUID

In all the aforementioned linear water wave studies, free surface-waves in a fluid of constant density over the entire fluid domain are considered. However, waves can also exist at the interface between two immiscible liquids of different densities. Such a sharp density gradient can, for example be generated in the ocean by solar heating of the upper layer, or in an estuary or a fjord into which fresh (less saline) river water flows over oceanic water, which is more saline and consequently heavier. The situation can be idealized as two-layer fluid by considering a lighter fluid of density ρ_1 lying over a heavier fluid of density ρ_2 . In the case of a two-layer fluid having an interface and a free surface, two different propagating modes may be excited during the wave motion. The waves generated due to the presence of the free surface are referred to as surface modes (SM) whilst the waves generated due to the presence of the interface are referred to as internal modes (IM) (see Milne-Thomson (1996) and Kundu and Cohen (2002)). The waves in both the modes propagate simultaneously in the two-layer fluid. This makes the mathematical formulation and analysis quite complex. This may be the probable reason for negligible progress in two-layer fluid studies. A review on linear water wave studies in a two-layer fluid is presented in this section subsequently.

The propagation of waves in a two-layer fluid with both a free surface and an interface (in the absence of any obstacles) was first investigated by Stokes (1847) and the classical problem of this type of two-layer fluid separated by a common interface with the upper fluid having a free surface is given in Lamb (1932) Art. 231 and Wehausen and Laitone (1960). Until recently, very little work has been done on wave/structure interaction in two-layer fluid. Sturova (1994) approximated the free surface as a rigid lid and studied the radiation of waves by an oscillating cylinder, moving uniformly in a direction perpendicular to its axis. Linton and McIver (1995) developed a general theory for two-dimensional wave scattering by horizontal cylinders in an infinitely deep two-layer fluid. They derived the reciprocity relations that exists between the various hydrodynamic characteristics of

the cylinders.

It is well-known that a circular cylinder submerged in an infinitely-deep uniform fluid reflects no wave energy, and it was shown in Linton and McIver (1995) that this is also true for a cylinder in the lower layer of a two-layer fluid. Zilman and Miloh (1995, 1996) analyzed the effect of a shallow layer of fluid mud on the hydrodynamics of floating bodies. Sturova (1999) considered the radiation and scattering problem for a cylinder both in a two-layer as well as in a three-layer fluid bounded above and below by rigid horizontal walls. For the three-layer case the middle layer was linearly stratified representing a smooth pycnocline. Using the method of multi modes Sturova was able to calculate the hydrodynamic characteristics of the cylinder. Gavrilov et al. (1999) investigated the effects of a smooth pycnocline on wave scattering for a horizontal circular cylinder where the fluid is bounded above and below by rigid walls. Their work included a comparison between theoretical and experimental results, with reasonable qualitative agreement but notable quantitative disagreement. A similar approach is to assume that the pycnocline is very thin and model the interface between the two fluids as a sharp discontinuity between layers of constant density. In the absence of obstacles, the appropriate dispersion relation for such a two-layer fluid has two solutions for a given frequency (Lamb (1932), Art. 231). One of these solution corresponds to waves where the majority of the disturbance is close to the free surface and the other to waves on the interface between the two fluid layers.

Work on three-dimensional scattering can be found in Yeung and Nguyen (1999) and Cadby and Linton (2000). In the former study, an integral equation technique was employed to solve radiation and diffraction problems for a rectangular barge in finite depth, whereas in the latter study, multi-pole expansions were used to solve problems involving submerged spheres in water of infinite depth. The symmetry relations for the added-mass and damping matrices and an analogue to the Haskind relations were given in Yeung and Nguyen (1999). Lee and Chwang (2000b) studied the wave transformation by a vertical barrier between a single-layer fluid and a two-layer fluid. By using the linear-wave theory and eigenfunction-expansions the boundary value problems are solved by a suitable application of the least-squares method. The definitions of the corresponding reflection and

transmission coefficients are introduced in each of the cases that they have considered. It is found that water waves, propagating either from the homogeneous or from the two-layer fluid, are partially reflected or transmitted and produced simultaneously in both modes (SM and IM) of water waves in the two-layer fluid.

In Barthélemy et al. (2000), the scattering of surface-waves by a step bottom in a two-layer fluid was considered. This problem is of particular interest to understand how tides are scattered at the continental shelf break. A WKBJ technique, which approximates the solution by simple traveling waves locally, was employed to find the reflection and transmission coefficients of the surface-waves past the step. A complete derivation of reciprocity relations for three-dimensional scattering in two-layer fluids can be found in Cadby and Linton (2000). The motivation for their work came from a plan to build an underwater pipe bridge across one of the Norwegian fjords. In the Norwegian fjords typically, bodies of waters consists of a layer of fresh water of about 10 *m* thick lies on the top of a very deep body of salt water. An extension model by Linton and Cadby (2002) to oblique waves of the work of Linton and McIver (1995) in which a linear water wave theory concerning the interaction of oblique waves with horizontal circular cylinder in either the upper or lower layer are also solved using multipole expansions. However, very little progress has been made on wave interaction with flexible/porous structures in two-layer fluid.

Ahmed (1998) considered the case of two immiscible layers of incompressible fluids in the presence of a porous wave maker immersed vertically in the two fluid, which are periodic in the horizontal direction direction along the wave maker as well as in time. An asymptotic analysis for large values of time and distance is given for the depression of the free surface and the surface of separation between the two fluids. His results indicate that there are two modes of waves spreading at each of the free surface and surface of separation. Also his results justify the use of Sommerfeld radiation condition at infinity when investigating steady state harmonic surface in wave problems. The application of this condition instead of boundedness condition at infinity is necessary to render the solution unique. Sherief et al. (2003) investigated the two-dimensional steady linear

gravity wave motion in two immiscible layers of incompressible and nonviscous fluid in the presence of a porous wave maker immersed vertically in the two fluids, where the surface of the upper fluid is free. Manam and Sahoo (2005) tackled analytically the problem of waves past rigid porous structures in two-layer fluid by making use of the generalized orthogonal relation. They obtained complete analytical solutions for the boundary value problems corresponding to the generation or scattering of axi-symmetric waves by two impermeable and permeable co-axial cylinders.

There are only a few studies on internal-wave trapping. Greenspan (1970) found solutions for coastal trapped surface- and internal-waves in a continuously and uniformly stratified fluid on a sloping beach.

2.4 MOTIVATION FOR THE PRESENT INVESTIGATION

The objective of the present investigation is to develop a simple, economic and accurate mathematical models for predicting the wave and structural behaviors for the wave structure interaction problems in a two-layer fluid. The investigation also aims at studying the performance of various submerged and floating breakwaters (rigid, flexible, porous, partial breakwaters) in attenuating the surface- and internal-waves in a two-layer fluid.

Until very recent years, the research works on wave-structure interaction problems in a two-layer fluid have been found very rarely. Again, the propagation of surface- and internal-waves in a two-layer fluid presents a challenge not only in developing a mathematical model but also in analyzing the complex flow physics. This is the reason why there is a negligible progress in the two-layer fluid studies. All these have prompted the present investigation.

Chapter 3

GENERAL MATHEMATICAL FORMULATION

3.1 INTRODUCTION

Knowledge of waves and the forces they generate is essential for the design of coastal projects since they are the major factors that determines the geometry of beaches, planning and design of marinas, waterways, shore protection measures, hydraulic structures, and other civil and military coastal works. Estimation of wave conditions are needed in almost all coastal engineering studies. To study the water wave propagation in the ocean over the years many theories were developed. The most elementary wave theory is the small-amplitude or linear wave theory. This theory, developed by Airy (1845), is easy to apply, and gives a reasonable approximation of wave characteristics for a wide range of wave parameters. Although there are limitations to its applicability, linear theory can still be useful provided the assumptions made in developing this simple theory are not grossly violated. The assumptions made in developing the linear wave theory applied in this work are:

1. The fluid is homogeneous and incompressible; therefore, the density is a constant.
2. Surface tension can be neglected.
3. Coriolis effect due to the earth's rotation can be neglected.

4. Pressure at the free surface is uniform and constant.
5. The fluid is ideal or inviscid (lacks viscosity) and the flow is irrotational.
6. The wave amplitude is small.
7. The seabed is horizontal, fixed and impermeable.
8. Waves are long-crested, i.e., two-dimensional.
9. The particular wave being considered does not interact with any other form of water motion like current.

The first three assumptions are valid for virtually all practical situations. It is necessary to relax the fourth, fifth and ninth assumptions for some specialized problems. Relaxing the sixth, seventh and eighth assumptions is also acceptable for many problems related to coastal engineering. In the present thesis linear wave theory is used to solve various wave-structure interaction problems in a two-layer fluid. Hence present mathematical model is based on all the above assumptions.

Mathematical modeling for solving wave-structure interaction problems in a two-layer fluid involves not only a particular way of writing the equations of fluid flow and structural motion but also selection of suitable numerical techniques to solve those equations. The purpose of this chapter is to introduce the mathematical model for fluid flow and structural response along with the various solution techniques that are used in the present thesis. In mathematical formulation and solution techniques the emphasis is given on those aspects that are specific to the problems dealing with fluid structure interaction in a two-layer fluid considered in the work.

3.2 MATHEMATICAL MODEL FOR TWO-LAYER FLUID

The simplest model of an internal-wave involves two layers of constant density fluids. Here the attention is restricted to the typical case of an ocean, fjord, or lake where the upper layer of density ρ_1 has a relatively light liquid and the lower layer of density ρ_2 is a

relatively heavy liquid (see Fig. 1.1 in Chapter 1). The general physical and mathematical model of the two-layer system is similar to that used in the analysis of water waves in a single-layer fluid. Because the densities are taken to be piecewise constant, the condition of incompressible flow is made in each liquid. Furthermore, the condition of irrotational, inviscid flow is also reasonable for each layer so that the governing equations in Fluid 1 and 2 can be taken to be Laplace's equation and the incompressible, irrotational, unsteady version of Bernoulli's equation.

The physical basis of the boundary conditions at the bottom and interface between Fluid 1 and 2 is also essentially the same as in the classical case. At a rigid bottom, only the kinematic boundary condition (KBC) is required. At the interfaces between the various fluids, both the kinematic and dynamic boundary conditions are required. Surface tension is normally negligible so that the dynamic boundary condition (DBC) reduces to the condition of zero pressure jump across the interfaces. An issue which is normally not faced in the study of ordinary water waves is that KBC imposes constraints on the velocities on both sides of the interface between Fluids 1 and 2. In a two-layer fluid, KBC must be applied explicitly on each side of the interface.

The analysis of the resultant system of equations is essentially the same as the classical water wave problem. In two-layer fluid dispersion relation the frequency can be found as a function of wavenumber (or equivalently, the wavelength), the thicknesses of each liquid layer, the ratio of the densities of fluids 1 and 2, and the acceleration of gravity. These roots of the dispersion relation represent right and left moving surface-like waves and right and left moving internal-waves. In the present section governing equations and boundary conditions for wave motion in a two-layer fluid are described subsequently.

3.2.1 Definition of Velocity Potential and Governing Equation

Consider a case where 2D wave propagate in the x direction, and y is the vertical coordinate, which is taken on -ve direction here for convenience (see Fig. 1.1). The free surface displacement is $\eta_{fs}(x, t)$. The fluid domain is assumed to be of uniform finite depth. According to the assumptions made earlier, a velocity potential $\Phi(x, y, t)$ exists

such that $\Phi(x, y, t) = \text{Re}[\phi_0 \phi(x, y) \exp(-i\omega t)]$, where ω is the wave frequency. The factor $\phi_0 = -igI_0/\omega$ is removed for convenience in the construction of velocity potentials containing the eigenfunctions, where I_0 is the amplitude of the incident waves and g is the gravitational constant. The Governing Equation for the problem is specified by Laplace equation for ϕ :

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0. \quad (3.1)$$

3.2.2 Linearized Free Surface Boundary Conditions

At the free surface, the KBC specify that the fluid particle never leaves the surface, that is

$$\frac{D\eta_{fs}}{Dt} = \phi_y, \quad \text{at } y = \eta_{fs}, \quad (3.2)$$

where $D/Dt = \partial/\partial t + U(\partial/\partial x)$ is the material or substantial derivative, and ϕ_y is the vertical component of fluid velocity at the free surface. Eq. 3.2 then can be written as:

$$\left(\frac{\partial \eta_{fs}}{\partial t} + U \frac{\partial \eta_{fs}}{\partial x} \right)_{y=\eta_{fs}} = \left(\frac{\partial \phi}{\partial y} \right)_{y=\eta_{fs}}. \quad (3.3)$$

For small-amplitude waves, both U and $\partial \eta_{fs}/\partial x$ are small, so that the quadratic term $U(\partial \eta_{fs}/\partial x)$ is one order smaller than other terms in Eq. 3.3, which then simplifies to

$$\left(\frac{\partial \eta_{fs}}{\partial t} \right)_{y=\eta_{fs}} = \left(\frac{\partial \phi}{\partial y} \right)_{y=\eta_{fs}}. \quad (3.4)$$

This condition can be simplified further by noting that the right hand side can be evaluated at $y = 0$ rather than at the free surface. To justify this, $\partial \phi/\partial y$ is expanded in a Taylor Series around $y = 0$.

$$\left(\frac{\partial \phi}{\partial y} \right)_{y=\eta_{fs}} = \left(\frac{\partial \phi}{\partial y} \right)_{y=0} + \left(\eta_{fs} \frac{\partial^2 \phi}{\partial y^2} + \dots \right)_{y=0}. \quad (3.5)$$

Therefore, to the first order of accuracy desired here, $\partial \phi/\partial y$ in Eq. 3.4 can be evaluated at $y = 0$. We then have

$$\frac{\partial \eta_{fs}}{\partial t} = \frac{\partial \phi}{\partial y}, \quad \text{at } y = 0. \quad (3.6)$$

In addition to the KBC at the free surface, there is a DBC that the pressure just below the free surface is always equal to the ambient pressure, with surface tension neglected.

Taking the ambient pressure to be zero, the surface boundary condition is

$$pr = 0, \quad \text{at} \quad y = \eta_{fs}. \quad (3.7)$$

Since the motion is irrotational, the following form of Bernoulli's equation is applicable:

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}(U^2 + V^2) + \frac{pr}{\rho} - gy = F(t), \quad (3.8)$$

where g is the gravitational constant. The negative sign appears in the body force term as the y coordinate is positive vertical in downward direction (see Fig. 1.1). The function $F(t)$ can be absorbed in $\partial\Phi/\partial t$ by redefining Φ and Eq. 3.8 can be simplified by neglecting nonlinear term $(U^2 + V^2)$ for small-amplitude waves, and the linearized form of the unsteady Bernoulli equation is

$$\frac{\partial \Phi}{\partial t} + \frac{pr}{\rho} - gy = 0. \quad (3.9)$$

Substituting into the surface boundary condition Eq. 3.7 in Eq. 3.9 gives

$$\frac{\partial \Phi}{\partial t} - g\eta_{fs} = 0, \quad \text{at} \quad y = \eta_{fs}. \quad (3.10)$$

As explained before, for small-amplitude waves, the term $\partial\Phi/\partial t$ can be evaluated at $y = 0$ rather than at $y = \eta_{fs}$ to give the following linear DBC:

$$\frac{\partial \Phi}{\partial t} - g\eta_{fs} = 0, \quad \text{at} \quad y = 0. \quad (3.11)$$

The KBC and DBC given by Eqs. 3.6 and 3.11 can be combined to give a single linear free surface condition:

$$\frac{\partial \phi}{\partial y} + K\phi = 0, \quad \text{on} \quad y = 0, \quad (3.12)$$

where $K = \omega^2/g$.

3.2.3 Linearized Interface Boundary Conditions

Following the similar approach as in the case of free surface boundary condition we can obtain the conditions at interface for small-amplitude waves. At the interface ($y = h$),

the continuity of the vertical component of velocity and pressure yield the boundary conditions (see Wehausen and Laitone (1960))

$$\left(\frac{\partial\phi}{\partial y}\right)_{y=h_+} = \left(\frac{\partial\phi}{\partial y}\right)_{y=h_-}, \quad \text{and} \quad (3.13)$$

$$\left(\frac{\partial\phi}{\partial y} + K\phi\right)_{y=h_+} = s\left(\frac{\partial\phi}{\partial y} + K\phi\right)_{y=h_-}, \quad (3.14)$$

where s is the density ratio in a two-layer fluid and is defined as $s = \rho_1/\rho_2$ with $0 < s < 1$.

3.2.4 Boundary Condition on the Rigid Boundaries

The condition on any rigid boundaries is given by

$$\frac{\partial\phi}{\partial n_i} = 0, \quad (3.15)$$

where n_i 's are the outward normals to the boundaries.

Boundary Condition on the Seabed

The boundary condition on the seabed ($y = H$) which is assumed horizontal, fixed and impermeable is given by:

$$\frac{\partial\phi}{\partial y} = 0, \quad \text{at } y = H. \quad (3.16)$$

3.2.5 Radiation Conditions at Infinity

In addition to all the above boundary conditions the velocity potential must satisfy the radiation conditions corresponding to reflected and transmitted waves. The radiation conditions for the wave motion in a two-layer fluid are given by

$$\phi \rightarrow \sum_{n=I}^{II} (I_n e^{ip_n x} + R_n e^{-ip_n x}) f_n(p_n, y) \quad \text{as } x \rightarrow -\infty, \quad (3.17)$$

and

$$\phi \rightarrow \sum_{n=I}^{II} T_n e^{ip_n x} f_n(p_n, y) \quad \text{as } x \rightarrow +\infty, \quad (3.18)$$

where I_I , R_I , T_I and I_{II} , R_{II} , T_{II} are the incident, reflected and transmitted wave amplitudes in surface mode (SM) and internal mode (IM) respectively. It may be noted that

p_I and p_{II} are wave numbers for the incident waves in SM and IM respectively. $f_n(p_n, y)$'s are the unknown eigenfunctions which have to be determined by solving the governing equation (Eq. 3.1) and imposing the appropriate boundary conditions according to the physical problem under consideration. Similar definitions for the velocity potentials in a scattering problem for a two-layer fluid are given by Barthélemy et al. (2000).

3.2.6 Continuity Conditions Across the Gap

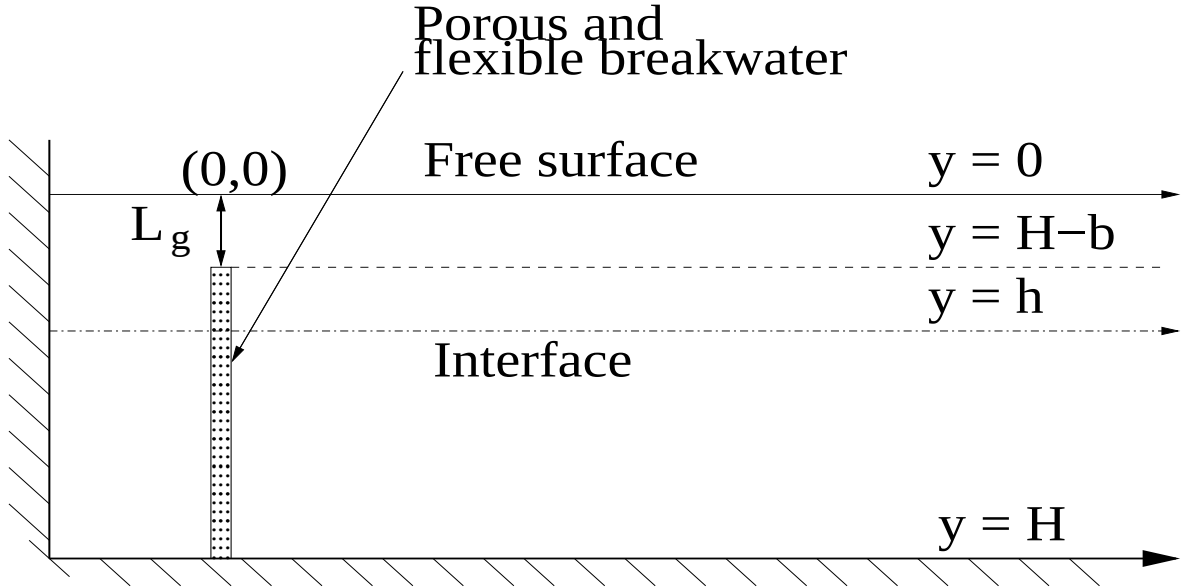


Figure 3.1: Definition sketch for gap in case of bottom-standing partial breakwater.

Across any imaginary boundary separating the two fluid regions (see, Figs. 3.1 and 3.2) over/underneath the breakwater (referred to as as gap, and the location is defined by $y \in L_g$) the continuity of velocity and pressure gives

$$\phi_1 = \phi_2 \text{ and } \frac{\partial \phi_1}{\partial x} = \frac{\partial \phi_2}{\partial x} \quad \text{on } y \in L_g. \quad (3.19)$$

Here ϕ_1 and ϕ_2 represent the upstream and downstream velocity potentials respectively across the gap.

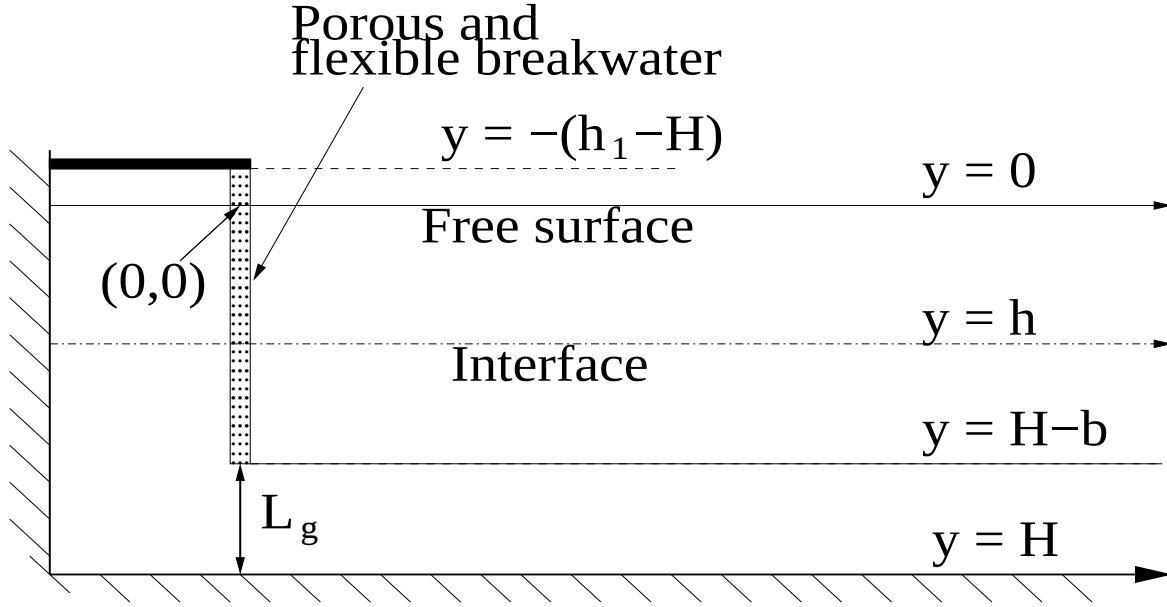


Figure 3.2: Definition sketch for gap in case of surface-piercing partial breakwater.

3.3 MATHEMATICAL MODEL FOR BREAKWATER RESPONSE

The vertical flexible breakwater response is analyzed by assuming that breakwater behaves like a one-dimensional beam of uniform flexural rigidity EI , axial force T and mass per unit length m_s . It is assumed that breakwater is deflected horizontally with displacement $\zeta(y, t) = \text{Re}[\xi(y)e^{-i\omega t}]$, where $\xi(y)$ represents the complex deflection amplitude and is assumed to be small as compared to the water depth. In the present thesis work both the cases of the plate and membrane breakwater are solved. In this section the governing equations and boundary conditions are presented for both plate and membrane breakwaters.

3.3.1 Governing Equation

For a Plate

The governing equation of the vertical plate breakwater response is given by

$$EI \frac{d^4 \xi}{dy^4} - m_s \omega^2 \xi = \begin{cases} 0, & \text{on } y \in L_{op}, \\ i\omega \rho_1 (\phi_1 - \phi_2), & \text{on } y \in L_{uf}, \\ i\omega \rho_2 (\phi_1 - \phi_2), & \text{on } y \in L_{lf}. \end{cases} \quad (3.20)$$

For a Membrane

The governing equation of the vertical membrane breakwater response is given by

$$T \frac{d^2 \xi}{dy^2} + m_s \omega^2 \xi = \begin{cases} 0, & \text{on } y \in L_{op}, \\ i\omega \rho_1 (\phi_1 - \phi_2), & \text{on } y \in L_{uf}, \\ i\omega \rho_2 (\phi_1 - \phi_2), & \text{on } y \in L_{lf}, \end{cases} \quad (3.21)$$

where L_{uf} and L_{lf} represent the parts of the breakwater in upper and lower fluid domain respectively. L_{op} represents the part of the breakwater above the free surface. ϕ_1 and ϕ_2 represent the upstream and downstream velocity potentials respectively across the breakwater.

3.3.2 Edge Conditions

For a Plate

At the clamped edge of the breakwater, the vanishing of the displacement and slope of deflection yield

$$(\xi)_{y=CE} = 0, \text{ and } (\xi')_{y=CE} = 0. \quad (3.22)$$

At the free edge, the vanishing of bending moment and shear force give rise to

$$(\xi'')_{y=FE} = 0, \text{ and } (\xi''')_{y=FE} = 0. \quad (3.23)$$

For a Membrane

At the clamped edge of the breakwater, the vanishing of the displacement yield

$$(\xi)_{y=CE} = 0. \quad (3.24)$$

At the free edge, the vanishing of slope of deflection give rise to

$$(\xi')_{y=FE} = 0, \quad (3.25)$$

where CE and FE represents the location of clamped and free edges of different breakwater configurations.

3.3.3 Continuity Condition Across the Free Surface

For different regions the breakwater response is defined as below (see, Figs. 3.1 and 3.2).

$$\xi(y) = \begin{cases} \xi_{op}(y), & \text{for } x = 0, y \in L_{op}, \\ \xi_{uf}(y), & \text{for } x = 0, y \in L_{uf}, \\ \xi_{lf}(y), & \text{for } x = 0, y \in L_{lf}. \end{cases} \quad (3.26)$$

For a Plate

Across the free surface ($y = 0$), the continuity of plate breakwater deflection, slope of plate breakwater deflection along with the bending moment and shear force acting on the plate breakwater (Yip et al. (2002)) yield

$$\xi_{op}(0) = \xi_{uf}(0), \quad \xi'_{op}(0) = \xi'_{uf}(0), \quad \xi''_{op}(0) = \xi''_{uf}(0), \quad \xi'''_{op}(0) = \xi'''_{uf}(0). \quad (3.27)$$

For a Membrane

Across the free surface ($y = 0$), the continuity of membrane breakwater deflection along with the slope of membrane breakwater deflection yield

$$\xi_{op}(0) = \xi_{uf}(0), \quad \xi'_{op}(0) = \xi'_{uf}(0). \quad (3.28)$$

3.3.4 Continuity Condition Across the Interface

For a Plate

Across the interface ($y = h$), the continuity of plate breakwater deflection, slope of plate breakwater deflection along with the bending moment and shear force acting on the plate

breakwater yield

$$\xi_{uf}(h) = \xi_{lf}(h), \quad \xi'_{uf}(h) = \xi'_{lf}(h), \quad \xi''_{uf}(h) = \xi''_{lf}(h), \quad \xi'''_{uf}(h) = \xi'''_{lf}(h). \quad (3.29)$$

For a Membrane

Across the interface ($y = h$), the continuity of membrane breakwater deflection along with the slope of membrane breakwater deflection yield

$$\xi_{uf}(h) = \xi_{lf}(h), \quad \xi'_{uf}(h) = \xi'_{lf}(h). \quad (3.30)$$

3.4 CONDITION ON POROUS AND FLEXIBLE BREAKWATER

In the present study the boundary condition on the vertical porous breakwaters is derived following the steps of Yu and Chwang (1994a), which is a generalization of the one developed by Chwang (1983). Here, the porous-effect parameter (Chwang (1983)) also called the Chwang parameter (Lee and Chwang (2000a)) is a complex number, which includes both the inertia and resistance effects.

Within the porous breakwater, the fluid flow is assumed to be followed by governing equations presented by Sollitt and Cross (1972) (see also Dalrymple et al. (1991)). As the breakwater is assumed to be flexible hence the equation of motion in the horizontal directions has the following form:

$$S\left(\frac{\partial U}{\partial t} + \frac{\partial^2 \zeta}{\partial t^2}\right) = -\frac{\nabla Pr}{\rho} - f\omega\left(U + \frac{\partial \zeta}{\partial t}\right), \quad (3.31)$$

where U is the time-dependent horizontal velocity vector of the fluid, $\zeta(y, t)$ is the time-dependent response of the flexible porous breakwater, ∇Pr is the hydrodynamic pressure gradient, ρ is the fluid density, f is the linearized resistance, ω is the angular frequency and $S = 1 + ((1 - \gamma)/\gamma)C_M$ is the coefficient of the inertial force acting on the porous medium with γ denoting the porosity and C_M the added mass coefficient. In the

present analysis it is assumed that ζ , U and Pr are sinusoidal with respect to time, i.e., $(\zeta, U, Pr) = (\xi, u, pr)e^{-i\omega t}$, and by applying this in Eq. 3.31 we obtain:

$$u = -\frac{\nabla pr}{\rho} \frac{1}{\omega(f - iS)} + i\omega\xi, \quad (3.32)$$

which indicates that the horizontal velocity is proportional to the pressure gradient.

The porous breakwater is assumed as a thin structure in the present study and hence, the variation of pressure across its thickness may be approximated as linear. The horizontal velocity component u within the porous medium can thus be expressed in terms of the pressure jump from one side of the porous wall to another, that is

$$u = \frac{pr_1 - pr_2}{\rho b \omega (f - iS)} + i\omega\xi, \quad (3.33)$$

where pr_1 and pr_2 represents the upstream and downstream pressure across the porous breakwater respectively and b is the physical thickness of the porous structure.

Since the linearized Bernoulli equation gives

$$\frac{pr}{\rho} = i\omega\phi, \quad (3.34)$$

and the continuity law across the breakwater leads to a relation

$$\gamma u_{1,2} = \frac{\partial\phi_1}{\partial x} = \frac{\partial\phi_2}{\partial x}, \quad (3.35)$$

we finally have

$$\frac{\partial\phi_{1,2}}{\partial x} = ik_0 G(\phi_1 - \phi_2) + i\omega\xi, \quad (3.36)$$

where k_0 is the incident wave number and

$$G = \frac{\gamma(f + iS)}{k_0 b (f^2 + S^2)} = G_r + iG_i, \quad (3.37)$$

is a complex porous-effect parameter as defined by Yu and Chwang (1994a). If the resistance is predominant in the porous medium, that is, $S \ll f$, $G = G_r = \gamma/(k_0 b f)$ is purely real. Eq. 3.36 then coincides with the formula used by Chwang (1983), which was based on the assumption that the flow in the porous medium is governed by Darcy's law and the porous structure is rigid. On the other hand, if the inertial effect in the

porous medium is more important, that is, $f \ll S$, we have $G = iG_i = i\gamma/(k_0 b S)$, which is a purely imaginary value. Under this circumstances, Eq. 3.36 is consistent with that derived by Macaskill (1979) on the basis that the Bernoulli equation holds for the porous medium flow. Generally, both the resistance and the inertial effect are important. The parameters f and S may be empirically evaluated following Madsen (1974). However, for accuracy hydraulic experiments are necessary to estimate the values of f and S for different breakwater configurations. Some results on the same can be found in Li (2006). In the present study above derived complex porous-effect parameter G is used in the boundary condition (Eq. 3.36) across the porous breakwater.

3.5 SOLUTION TECHNIQUES

To solve any fluid structure interaction problem, selection of proper solution techniques and computational tools suiting the problem under consideration is one of the major task. In the present work, the following three solution techniques are utilized for solving the aforementioned boundary value problems.

3.5.1 Eigenfunction-Expansion Method

The governing equation for all problems considered here is the Laplace equation. Solutions of Laplace equation are called harmonic functions. Apart from water wave modeling Laplace equation finds its application in many areas like heat conduction problems, mechanics, electromagnetism, probability, quantum mechanics, gravity and biology. The eigenfunction-expansion method is a powerful computational tool for solving boundary value problems governed by Laplace equation. Eigenfunction-expansion method is particularly being utilized extensively in the water wave modeling for past several decades. Generally in a water wave modeling problems this method is used to construct the velocity potentials, which also provides the dispersion relation. In general dispersion relations have more than one solutions and these solutions are known as eigenvalues. Depending upon the physics of problem, necessary eigenvalues are included in the expression of the

velocity potential. The functions in the velocity potential that contain these eigenvalues are known as eigenfunctions. Eigenfunction-expansion method is utilized to solve different physical problems considered in the subsequent chapters of the present work.

3.5.2 Wide-Spacing-Approximation Method (WSAM)

In general wave scattering by multiple bodies is a complex problem. To simplify the problem in the past many researchers have utilized the wide-spacing-approximation method (WSAM) to solve multi-body problems. In this method it is assumed that the only interactions occur arise from plane waves traveling between the bodies. Accuracy of this type of approximation has been demonstrated by a number of authors for several cases (Srokosz and Evans (1979), and Martin (1984)). The basic assumptions in a WSAM is that both the wavelength and a typical body dimension must be much less than the separation distance between the bodies (Martin (1984)). It is quite interesting to note that WSAM has consistently provided good results even though these assumptions are clearly violated in many situations. In the present study WSAM is utilized to solve the multi-body problems and the results are compared with those obtained with eigenfunction-expansion method.

3.5.3 Least-Squares-Approximation Method

In general for potential flow problems in water wave modeling equations with series expansion in orthogonal polynomials are encountered. For such equations the evaluation of coefficients by least-squares methods is significantly simpler. The values of the expansion coefficients are independent of the point of truncation of the series. For example consider a series expression obtained in the form

$$\sum_{n=1}^{\infty} R_n f_n(y) = 0, \text{ for a domain,} \quad (3.38)$$

where R_n and $f_n(y)$ are complex coefficients and functions with complex variables respectively. We can then write

$$Q(y) = \sum_{n=1}^N R_n f_n(y). \quad (3.39)$$

Applying the least-squares method and integrating over the defined domain we can obtain

$$\int_{LL}^{UL} |Q(y)|^2 dy = \text{minimum.} \quad (3.40)$$

where UL and LL represents the upper and lower limit for the integral. Minimizing the above integral with respect to each R_n 's yields

$$\int_{LL}^{UL} \bar{Q}(y) Q^* dy = 0, \quad (3.41)$$

where $Q^*(y) = \frac{\partial Q(y)}{\partial R_m}$, $\bar{Q}(y) = \sum_{n=1}^N \bar{R}_n \bar{f}_n(y)$ with bar denoting the complex conjugate and $R_m = R_1, R_2, \dots, R_N$. Least-squares method for the complex coefficients is expressed as above is also known as conjugate-gradient method. The expression (3.41) provides N linear equations with N unknowns, which can be solved easily by Gauss-elimination method. N can be selected by satisfying a convergence criteria based on the desired order of accuracy.

In the case of flexible breakwaters the number of unknowns are more than that of rigid structures. Numerical convergence problem is observed, when eigenfunction expansion method is employed directly for flexible breakwaters. Hence, in the present study the least-squares method is employed to solve the problems dealing with flexible breakwaters.

Chapter 4

WAVE SCATTERING BY SURFACE-PIERCING DIKES

4.1 INTRODUCTION

In the present chapter wave interaction with rigid non-porous structures of standard geometry in a two-layer fluid is considered. The case of surface-piercing dikes are analyzed and the results are compared with those, that exist in the literature for the same problem in a single-layer fluid. Wave scattering by such floating structures are considered in past by many researches within the context of linearized-theory of water waves. Some of the important studies in the context includes Mei and Black (1969) and McIver (1986). The mathematical formulation, solution techniques and numerical results for the scattering of surface- and internal-waves by a single and a pair of surface-piercing dikes in a two-layer fluid are presented here.

Cartesian co-ordinate system is chosen with the positive x -axis in the direction of wave propagation and positive y -axis in the downward direction. The dikes are approximated as cylinders of rectangular cross section and are placed in a two-layer fluid of finite depth. These rectangular dikes are partially immersed in a two-layer fluid. The problem is considered in two-dimensions under the assumptions that the dikes have parallel generators and are of sufficient length for normal waves incidence. The upper fluid has a free

surface (undisturbed free surface located at $y = 0$) and the two fluids are separated by a common interface (undisturbed interface located at $y = h$), each fluid is of infinite horizontal extent. The upper fluid of density ρ_1 occupies the region $-\infty < x < \infty$; $0 < y < h$ and the lower fluid of density ρ_2 occupy the region $-\infty < x < \infty$; $h < y < H$ in. The flow is simple harmonic in time with angular frequency ω . Thus the velocity potential $\Phi(x, y, t)$ can be expressed as: $\Phi(x, y, t) = \text{Re}[\phi(x, y)\exp(-i\omega t)]$.

The spatial velocity potential ϕ will satisfy the Laplace equation (Eq. 3.1) along with the appropriate boundary conditions (linearized free surface boundary condition Eq. 3.12, linearized interface boundary conditions Eqs. 3.13 and 3.14, boundary conditions on the rigid boundaries Eq. 3.15 and seabed condition Eq. 3.16), which depends on the region of the fluid under consideration. In addition radiation conditions at infinity must be imposed for uniqueness of the solution.

4.2 MODEL IN THE CASE OF A SINGLE DIKE

4.2.1 Definition of the Physical Problem

In this subsection, solution of wave scattering by a single surface-piercing dike is analyzed which is further generalized to study the wave scattering by a pair of identical surface-piercing dikes in the subsequent section. Wave scattering by a single surface-piercing dike as shown in Fig. 4.1 is considered, which is of width $2a$ and mean wetted draft d . The origin $(0, 0)$ is chosen to be the point where the mean free surface intersects with the vertical axis passing through the center of the dike.

The symmetry of the configuration can be exploited to simplify the solution by writing the velocity potential $\phi(x, y)$ as a sum of symmetric and anti-symmetric parts (similar to McIver (1986)) as given by

$$\phi = \phi^s + \phi^a, \quad (4.1)$$

where the symmetric velocity potential, ϕ^s and anti-symmetric velocity potential, ϕ^a are even and odd functions of x respectively. With this decomposition of velocity potential,

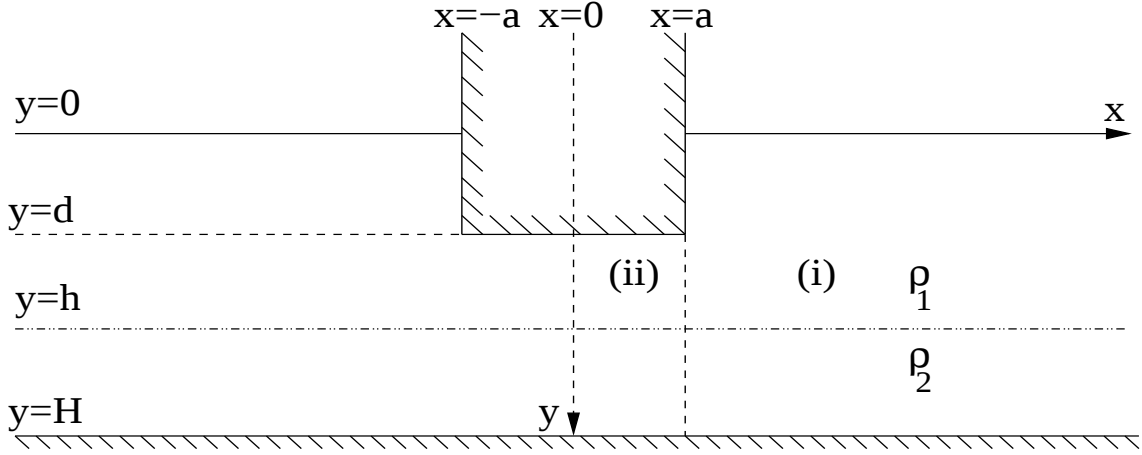


Figure 4.1: Definition sketch for single surface-piercing dike.

the boundary value problem reduces to a simpler problem in the region $x \geq 0$ only, and in addition the number of unknowns to be determined by solving a linear matrix system is also dramatically reduced.

4.2.2 Velocity Potentials

Considering the waves incident from large positive x upon the dike, the velocity potentials are obtained by eigenfunction-expansion method in each of the two regions (i) and (ii) as marked in Fig. 4.1. This is a boundary value problem specified by equation (3.1) along with the conditions, Eqs. (3.12), (3.13, 3.14, 3.15 and 3.16). Let us take $\phi(x, y)$ as

$$\phi(x, y) = X(x)Y(y). \quad (4.2)$$

Substituting $\phi(x, y)$ in Eq. (3.1) the problem reduces to

$$\frac{d^2 X}{dx^2} = p^2 X; \quad \text{and} \quad \frac{d^2 Y}{dy^2} = p^2 Y. \quad (4.3)$$

Considering the three different possibilities $p < 0$, $p = 0$ and $p > 0$ and suitably applying the boundary conditions we obtain the velocity potentials for different regions (regions (i) and (ii)). The velocity potentials are presented in symmetric and anti-symmetric parts in the subsequent subsections.

The expression for symmetric velocity potential ϕ^s in an open water region is given by

$$\phi_1^s = \frac{1}{2} \sum_{n=I}^{II} \exp(-ip_n(x-a))f_n(y) + \sum_{n=I,II,1}^{\infty} A_n^s \exp(ip_n(x-a))f_n(y), \text{ in region (i).} \quad (4.4)$$

The corresponding expression for the anti-symmetric velocity potential ϕ^a is given by

$$\phi_1^a = \frac{1}{2} \sum_{n=I}^{II} \exp(-ip_n(x-a))f_n(y) + \sum_{n=I,II,1}^{\infty} A_n^a \exp(ip_n(x-a))f_n(y), \text{ in region (i).} \quad (4.5)$$

In the open water region (i), the vertical eigenfunctions f_n 's, satisfying governing equation (3.1) along with the conditions Eqs. (3.12, 3.13, 3.14) and Eq. (3.16) on the seabed $y = H$, are given by

$$f_n(y) = \begin{cases} \frac{-N_n^{-1} \sinh p_n(H-h) [p_n \cosh p_n y - K \sinh p_n y]}{p_n \sinh p_n h - K \cosh p_n h}, & \text{for } 0 < y < h, \\ N_n^{-1} \cosh p_n(H-y), & \text{for } h < y < H, \end{cases} \quad (n = I, II, 1, 2, 3, \dots) \quad (4.6)$$

where

$$N_n^2 = (4p_n)^{-1} (K \cosh p_n h - p_n \sinh p_n h)^{-2} [(K \cosh p_n h - p_n \sinh p_n h)^2 (2(H-h)p_n + \sinh 2p_n(H-h)) - s \sinh^2 p_n(H-h) (K^2(2p_n h - \sinh 2p_n h) - p_n^2(2p_n h + \sinh 2p_n h) + 2Kp_n(\cosh 2p_n h - 1))]. \quad (4.7)$$

In the above p_I and p_{II} are positive real roots and p_n for $n \geq 1$ are positive purely imaginary roots of the dispersion relation in p given by

$$(1-s)p^2 \tanh p(H-h) \tanh ph - pK[\tanh ph + \tanh p(H-h)] + K^2[s \tanh p(H-h) \tanh ph + 1] = 0. \quad (4.8)$$

Here p_I and p_{II} represent the propagative modes for the surface- and internal-waves respectively.

The expression for symmetric velocity potential ϕ^s in the dike covered region is given by

$$\phi_2^s = B_0^s \chi_0(y) + \sum_{n=I,1}^{\infty} B_n^s \frac{\cos q_n x}{\cos q_n a} \chi_n(y), \text{ in region (ii).} \quad (4.9)$$

The corresponding expression for the anti-symmetric velocity potential ϕ^a is given by

$$\phi_2^a = \frac{B_0^a x \chi_0(y)}{a} + \sum_{n=I,1}^{\infty} B_n^a \frac{\sin q_n x}{\sin q_n a} \chi_n(y), \text{ in region (ii).} \quad (4.10)$$

For this dike covered region (ii), the eigenfunctions satisfying Eqs. (3.1, 3.13, 3.14 and 3.16) and Eq. (3.15) at the bottom of the dike ($y = d$), are given by

$$\chi_0(y) = \begin{cases} L_0^{-1}, & \text{for } d < y < h, \\ s L_0^{-1}, & \text{for } h < y < H, \end{cases} \quad (4.11)$$

and

$$\chi_n(y) = \begin{cases} \frac{-L_n^{-1} \sinh q_n(H-h) \cosh q_n(y-d)}{\sinh q_n(h-d)}, & \text{for } d < y < h, \\ L_n^{-1} \cosh q_n(H-y), & \text{for } h < y < H, \end{cases} \quad (n = I, 1, 2, 3, \dots) \quad (4.12)$$

where

$$L_0^2 = s(s(H-h) + (h-d)), \quad (4.13)$$

and

$$L_n^2 = (4q_n)^{-1} (\sinh q_n(h-d))^{-2} [s \sinh^2 q_n(H-h)(2(h-d)q_n + \sinh 2q_n(h-d)) + \sinh^2 q_n(h-d)(2(H-h)q_n + \sinh 2q_n(H-h))]. \quad (4.14)$$

In the above q_I is positive real and q_n 's for $n \geq 1$ are positive purely imaginary roots of the dispersion relation in q as given by

$$(1-s)q \tanh q(H-h) \tanh q(h-d) - K[\tanh q(h-d) + s \tanh q(H-h)] = 0. \quad (4.15)$$

It may be noted that in the dike covered region, in case of $d < h$, the vertical eigenfunctions have only one propagating mode because of the presence of the interface, whereas in case of $d \geq h$ (the special case of surface obstacle), the vertical eigenfunctions χ_n 's do not satisfy the interface conditions Eqs. (3.13 and 3.14). Like in a single-layer fluid, in this later situation no propagative mode exists. Hence, similar to the case of a single-layer fluid problem the term $n = I$ does not appear in the expressions of eigenfunctions and the corresponding vertical eigenfunctions becomes

$$\chi_n = L_n^{-1} \cos q_n(H-y), \quad (n = 0, 1, 2, \dots), \quad (4.16)$$

where

$$L_0^2 = 1 - d/H, \text{ and } L_n^2 = 0.5(1 - d/H), \text{ for } (n \geq 1). \quad (4.17)$$

q_n satisfy the relation

$$q_n = n\pi/(H - d), \text{ for } n = 0, 1, 2, \dots \quad (4.18)$$

The radiation conditions which physically states that scattered wave must be out going from the dike require that (Barthélemy et al. (2000))

$$\phi \sim \sum_{n=I}^{II} [\exp(-ip_n x) f_n(y) + (A_n^s + A_n^a) \exp(-2ip_n a) \exp(ip_n x)] f_n(y) \text{ as } x \rightarrow \infty, \quad (4.19)$$

and

$$\phi \sim \sum_{n=I}^{II} (A_n^s - A_n^a) \exp(-2ip_n a) \exp(-ip_n x) f_n(y) \text{ as } x \rightarrow -\infty. \quad (4.20)$$

It may be noted that $A_n^{s,a} = 0.5R_n^{s,a}$ for $n = I, II$, where $R_I^{s,a}$ and $R_{II}^{s,a}$ are related to the reflection/transmission coefficients in SM and IM respectively. The expression for respective reflection and transmission coefficients in SM and IM (Manam and Sahoo (2005)) are given below.

The reflection and transmission coefficients in the SM are given by

$$Kr_I = \frac{1}{2} |(R_I^s + R_I^a) \exp(-2ip_I a)|, \quad Kt_I = \frac{1}{2} |(R_I^s - R_I^a) \exp(-2ip_I a)|. \quad (4.21)$$

The reflection and transmission coefficients in the IM are given by

$$Kr_{II} = \frac{1}{2} |(R_{II}^s + R_{II}^a) \exp(-2ip_{II} a)|, \quad Kt_{II} = \frac{1}{2} |(R_{II}^s - R_{II}^a) \exp(-2ip_{II} a)|. \quad (4.22)$$

On the solid boundaries, Eq. 3.15 must be satisfied. Hence the condition on wetted draft of the dike is given by

$$\frac{\partial \phi_1^{s,a}}{\partial x} = 0, \text{ for } 0 < y < d. \quad (4.23)$$

The condition across the gap Eq. 3.19 must be imposed to determine the unknowns $\{A_n^s, A_n^a, n = I, II, 1, \dots\}$ and $\{B_n^s, B_n^a, n = 0, I, 1, \dots\}$. Hence the continuity of pressure and horizontal velocity at $x = a$, give

$$\phi_1^{s,a} = \phi_2^{s,a}, \text{ for } d < y < H, \quad (4.24)$$

$$\frac{\partial \phi_1^{s,a}}{\partial x} = \frac{\partial \phi_2^{s,a}}{\partial x}, \quad \text{for } d < y < H. \quad (4.25)$$

In case of a two-layer fluid, the eigenfunctions do not follow the usual orthonormal relation as in the case of a single-layer fluid. In the present study a general orthonormal relation suitable for a two-layer fluid is used for both the open water and dike covered region eigenfunctions.

The eigenfunctions f_m 's are integrable in $0 < y < H$ having a single discontinuity at $y = h$, and are orthonormal (f_m 's are normalized eigenfunctions hence the orthogonal relation becomes orthonormal) with respect to the inner product as given below (see Manam and Sahoo (2005)).

$$\langle f_n, f_m \rangle_1 = s \int_0^h f_n f_m dy + \int_h^H f_n f_m dy, \quad (4.26)$$

Similar to f_m 's, χ_n 's are also orthonormal with respect to the inner product

$$\langle \chi_n, \chi_m \rangle_2 = \begin{cases} s \int_d^h \chi_n \chi_m dy + \int_h^H \chi_n \chi_m dy, & \text{for } d < h, \\ \int_d^H \chi_n \chi_m dy, & \text{for } d \geq h. \end{cases} \quad (4.27)$$

4.2.3 General Solution Procedure

Using the expressions (4.4) and (4.9) in the Eq. (4.24) and exploiting the orthonormality of the eigenfunctions χ_m 's as defined in relation (4.27), we obtain

$$B_m^s = \frac{1}{2} \sum_{n=I}^{II} C_{nm} + \sum_{n=I, II, 1}^{\infty} A_n^s C_{nm}, \quad m = 0, I, II, 1, \dots, \quad (4.28)$$

where, $C_{nm} = \langle f_n, \chi_m \rangle_2$. Using expressions (4.4) and (4.9) in Eq. (4.25) and exploiting the orthonormality of the eigenfunctions f_m 's as defined in relation (4.26), we obtain

$$\frac{1}{2} i \sum_{n=I}^{II} p_n \delta_{nm} - i p_m A_m^s = \sum_{n=0, I, 1}^{\infty} B_n^s q_n C_{mn} \tan q_n a, \quad m = I, II, 1, \dots, \quad (4.29)$$

where δ_{nm} is the Kronecker delta. Substituting B_n^s from Eq. (4.28) in (4.29) yields

$$i p_m A_m^s + \sum_{n=I, II, 1}^{\infty} \alpha_{mn}^s A_n^s = \frac{1}{2} \sum_{r=I}^{II} (i p_r \delta_{mr} - \alpha_{mr}^s), \quad m = I, II, 1, \dots, \quad (4.30)$$

where

$$\alpha_{mn}^s = \sum_{r=I,1}^{\infty} C_{mr} C_{nr} q_r \tan q_r a. \quad (4.31)$$

A similar matching procedure on $x = a$ for the anti-symmetric velocity potential ϕ^a gives

$$ip_m A_m^a - \sum_{n=I,II,1}^{\infty} (\alpha_{mn}^a + \gamma_{mn}) A_n^a = \frac{1}{2} \sum_{r=I}^{II} (ip_r \delta_{mr} + \gamma_{mr} + \alpha_{mr}^a), \quad m = I, II, 1, \dots, \quad (4.32)$$

where

$$\alpha_{mn}^a = \sum_{r=I,1}^{\infty} C_{mr} C_{nr} q_r \cot q_r a, \quad (4.33)$$

and

$$\gamma_{mn} = \frac{C_{m0} C_{n0}}{a}. \quad (4.34)$$

The systems of equations (4.30) and (4.32) are solved using Gauss-elimination method to obtain the various physical quantities of interest. These are found to have excellent convergence characteristics. In the computation number of evanescent modes in the series are selected based on the experience of the numerical convergence experiment. In numerical convergence experiment a study is carried out to estimate the number of evanescent modes N needed for convergence of the system of equation in the present wave structure interaction problems. It is observed that 8-10 evanescent modes are enough to obtain the results accurately up to 3-decimal point in most of the physical situation considered in the present study. A case study is plotted for wave past porous plate breakwater discussed in Fig. 7.2 (a and b) of Chapter 7. For computation of all numerical results 15 evanescent modes are considered.

4.3 MODEL IN THE CASE OF A PAIR OF IDENTICAL DIKES

4.3.1 Definition of the Physical Problem

In this section, solution for scattering of incident wave trains by a pair of identical surface-piercing dikes in a two-layer fluid is considered. The wave scattering by a pair of identical surface-piercing dikes is shown in Fig. 4.2 is solved, which are again of width $2a$ and draft d . The origin $(0, 0)$ is chosen to be at the center of the right-hand dike.

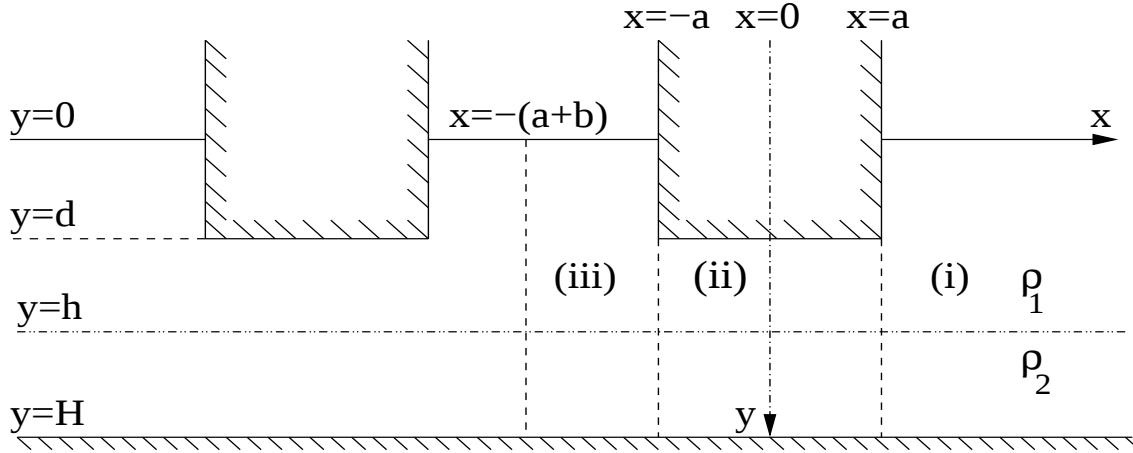


Figure 4.2: Definition sketch for a pair of identical surface-piercing dikes.

4.3.2 Velocity Potentials

The symmetry of the problem is again exploited by writing the solution as the sum of a symmetric and an anti-symmetric parts. The line of symmetry for present problem is $x = -(a + b)$ (see Fig. 4.2). Hence similar to a single surface-piercing dike we can write $\phi(x, y)$ as a sum of symmetric and anti-symmetric parts as in Eq. (4.1). With this decomposition of velocity potential, the boundary value problem reduces to a simpler problem in the region $x \geq -(a + b)$ only. In addition this decomposition of velocity potentials helps in increasing the computational efficiency by reducing the number of unknown coefficients.

The eigenfunction-expansion method similar to the one used in the case of a single surface-piercing dike is adopted in the present problem also to construct the velocity potentials. There are now two open water regions (region (i) and (iii)) and two matching boundaries. The appropriate eigenfunction-expansions for the symmetric velocity potentials in each region are:

$$\phi_1^s = \frac{1}{2} \sum_{n=I}^{II} \exp(-ip_n(x-a)) f_n(y) + \sum_{n=I,II,1}^{\infty} A_n^s \exp(ip_n(x-a)) f_n(y), \quad (4.35)$$

and

$$\phi_3^s = \sum_{n=I,II,1}^{\infty} D_n^s \frac{\cos p_n(x+a+b)}{\sin p_nb} f_n(y). \quad (4.36)$$

The appropriate eigenfunction-expansions for the anti-symmetric velocity potentials in each region are:

$$\phi_1^a = \frac{1}{2} \sum_{n=I}^{II} \exp(-ip_n(x-a)) f_n(y) + \sum_{n=I,II,1}^{\infty} A_n^a \exp(ip_n(x-a)) f_n(y), \quad (4.37)$$

and

$$\phi_3^a = \sum_{n=I,II,1}^{\infty} D_n^a \frac{\sin p_n(x+a+b)}{\cos p_nb} f_n(y). \quad (4.38)$$

The appropriate eigenfunction-expansions for the symmetric velocity potentials in dike covered region (region (ii)) is given by

$$\phi_2^s = [B_0^s + \frac{x}{a} C_0^s] \chi_0(y) + \sum_{n=I,1}^{\infty} [B_n^s \frac{\cos q_n x}{\cos q_n a} + C_n^s \frac{\sin q_n x}{\sin q_n a}] \chi_n(y). \quad (4.39)$$

The appropriate eigenfunction-expansions for the anti-symmetric velocity potentials in dike covered region (region (ii)) is given by

$$\phi_2^a = [B_0^a + \frac{x}{a} C_0^a] \chi_0(y) + \sum_{n=I,1}^{\infty} [B_n^a \frac{\cos q_n x}{\cos q_n a} + C_n^a \frac{\sin q_n x}{\sin q_n a}] \chi_n(y), \quad (4.40)$$

The vertical eigenfunctions and dispersion relations in open water, and dike covered regions are the same as derived in the case of a single surface-piercing dike. The radiation condition, definition of reflection and transmission coefficients, condition on the wetted draft of the dike, continuity condition across the gap are also similar to the case of a single surface-piercing dike. Moreover, the orthonormality conditions as defined in the case of a single surface-piercing dike is also applicable in the present problem.

4.3.3 General Solution Procedure

The matching procedure for $x = \pm a$ is very similar to that used for the single surface-piercing dike case. The resulting equations for the unknown coefficients are

$$p_m(iA_m^s - D_m^s) - \sum_{n=I,II,1}^{\infty} (\alpha_{mn}^a + \gamma_{mn})(A_n^s - D_n^s \cot p_n b) = \frac{1}{2} \sum_{r=I}^{II} (ip_r \delta_{mr} + \gamma_{mr} + \alpha_{mr}^a), \quad m = I, II, 1, \dots, \quad (4.41)$$

and

$$p_m(iA_m^s + D_m^s) + \sum_{n=I,II,1}^{\infty} \alpha_{mn}^s (A_n^s + D_n^s \cot p_n b) = \frac{1}{2} \sum_{r=I}^{II} (ip_r \delta_{mr} - \alpha_{mr}^s), \quad m = I, II, 1, \dots, \quad (4.42)$$

where $\alpha_{mn}^{s,a}$ and γ_{mn} are the same as defined in Eqs. (4.31, 4.33 and 4.34). The remaining coefficients, for the velocity potential ϕ_2^s (4.39), are given by

$$2B_m^s = \frac{1}{2} \sum_{n=I}^{II} C_{nm} + \sum_{n=I,II,1}^{\infty} C_{nm} (A_n^s + D_n^s \cot p_n b), \quad m = 0, I, 1, \dots, \quad (4.43)$$

and

$$2C_m^s = \frac{1}{2} \sum_{n=I}^{II} C_{nm} + \sum_{n=I,II,1}^{\infty} C_{nm} (A_n^s - D_n^s \cot p_n b), \quad m = 0, I, 1, \dots. \quad (4.44)$$

where $C_{nm} = < f_n, \chi_m >_2$.

The final equations for the anti-symmetric velocity potential coefficients $\{A_n^a, D_n^a; n = I, II, 1, \dots\}$ and $\{B_n^s, C_n^a; n = 0, I, 1, \dots\}$ are similar in form to Eqs. (4.41) – (4.44) except that $\cot p_n b$ must be replaced by $\tan p_n b$ throughout. This gives

$$p_m(iA_m^a - D_m^a) - \sum_{n=I,II,1}^{\infty} (\alpha_{mn}^a + \gamma_{mn})(A_n^a - D_n^a \tan p_n b) = \frac{1}{2} \sum_{r=I}^{II} (ip_r \delta_{mr} + \gamma_{mr} + \alpha_{mr}^a), \quad m = I, II, 1, \dots, \quad (4.45)$$

and

$$p_m(iA_m^a + D_m^a) + \sum_{n=I,II,1}^{\infty} \alpha_{mn}^s (A_n^a + D_n^a \tan p_n b) = \frac{1}{2} \sum_{r=I}^{II} (ip_r \delta_{mr} - \alpha_{mr}^s), \quad m = I, II, 1, \dots. \quad (4.46)$$

The remaining coefficients, for the velocity potential ϕ_2^a (Eq. 4.40), are given by

$$2B_m^a = \frac{1}{2} \sum_{n=I}^{II} C_{nm} + \sum_{n=I,II,1}^{\infty} C_{nm} (A_n^a + D_n^a \tan p_n b), \quad m = 0, I, 1, \dots, \quad (4.47)$$

and

$$2C_m^a = \frac{1}{2} \sum_{n=I}^{II} C_{nm} + \sum_{n=I,II,1}^{\infty} C_{nm} (A_n^a - D_n^a \tan p_n b), \quad m = 0, I, 1, \dots. \quad (4.48)$$

The equation sets (4.41) and (4.42) are solved simultaneously for unknown coefficients $\{A_n^s, D_n^s, n = I, II, 1, \dots\}$. The equation sets (4.45) and (4.46) are solved simultaneously to obtain the unknowns $\{A_n^a, D_n^a, n = I, II, 1, \dots\}$. Again Gauss-elimination method is used to solve the matrix systems and evanescent modes in the series are selected based on the experience of the numerical convergence experiment.

4.4 MODEL USING WSAM

Finding the solution for more than two dikes, or for multi dikes of different geometry, by an extension of the previously described methods is a non-trivial task. However, for a given characteristics of single dikes in isolation, it is possible to obtain approximate solutions for multi body problems using the WSAM. The distance between dikes is assumed to be large enough as compared to the wavelength of incident wave trains to neglect local effects while considering the interactions between the dikes. Hence, the only interactions will result from the plane waves propagating between the dikes. The results derived using WSAM for the case of two dikes and the extension to a large number of dikes is straightforward. The solution procedure in the present section is similar to that of McIver (1986), which was described in Srokosz and Evans (1979).

4.4.1 Solution Procedure Using WSAM

Consider a wave incident from large positive x upon two dikes centered on the points $x = c_1$ and $x = c_2$, $c_1 > c_2$. Far from the bodies, the spatial velocity potential $\phi(x, y)$ can

be written as

$$\phi_1 = \sum_{n=I}^{II} \left(\exp(-ip_n x) + R_n \exp(ip_n x) \right) f_n(y), \quad \text{for } x > c_1, \quad (4.49)$$

$$\phi_2 = \sum_{n=I}^{II} \left(A_n \exp(-ip_n x) + B_n \exp(ip_n x) \right) f_n(y), \quad \text{for } c_2 < x < c_1, \quad (4.50)$$

and

$$\phi_3 = \sum_{n=I}^{II} T_n \exp(-ip_n x) f_n(y), \quad \text{for } x < c_2. \quad (4.51)$$

where R_n and T_n for $n = I, II$, are related with the reflection and transmission coefficients for the dike pair. A_n and B_n for $n = I, II$, are the amplitude of the wave in central region propagating away from dike 1 and 2 respectively.

The total reflected wave results from the reflection of the incident wave by dike 1 and the transmission of the second component of ϕ_2 through dike 1, hence

$$R_m = R_{m1} \exp(-2ip_m c_1) + T_{m1} B_m, \quad \text{for } m = I, II. \quad (4.52)$$

where R_{mn} and T_{mn} , for $m = I, II$ are related with the reflection and transmission coefficients of dike n when in isolation, as defined in the case of a single surface-piercing dike. Following similar arguments

$$A_m = T_{m1} + R_{m1} B_m \exp(2ip_m c_1), \quad \text{for } m = I, II, \quad (4.53)$$

$$B_m = R_{m2} A_m \exp(-2ip_m c_2), \quad \text{for } m = I, II, \quad (4.54)$$

and

$$T_m = T_{m2} A_m, \quad \text{for } m = I, II. \quad (4.55)$$

From Eqs. (4.52) – (4.55), the explicit expressions for R_m and T_m in terms of R_{mn} and T_{mn} , for $m = I, II$ and $n = 1, 2$ are obtained.

4.5 NUMERICAL RESULTS AND DISCUSSION

Numerical results are computed and analyzed for surface- and internal-wave scattering by a single and a pair of identical dikes in a two-layer fluid. The effects of various non-dimensional physical parameters on wave reflection (in both SM and IM) and hydrodynamic forces experienced by the dikes are analyzed. For convenience, the wave parameters are given in terms of the non-dimensional wave number $p_I d$, gap between the dikes $p_I b$, depth ratio h/H , fluid density ratio s along with the non-dimensional dike parameters given by a/d , H/d and b/H .

4.5.1 Reflected Energy

Here we consider wave reflection by a single and a pair of identical surface-piercing dikes in both SM and IM. Results are plotted by allowing $p_{II} d$ (normalized wave number in IM) to vary based on the two-layer fluid dispersion relation.

In Fig. 4.3, present results for single surface-piercing dike reflection coefficients (Kr_I , Kr_{II} as defined in Eqs. (4.21) and (4.22)) in a two-layer fluid for different values of H/d ratios are compared with the results obtained by Mei and Black (1969) in a single-layer fluid. In general, the reflection coefficients in SM, Kr_I are similar to that observed by Mei and Black (1969) except in case of intermediate frequency range, where Kr_I is found to be significantly small. It may be noted that in case of a two-layer fluid, due to the presence of the interface, waves in SM and IM propagate below the dike (when $d < h$), which is not the case for single-layer fluid. For both small (corresponds to long wave region and the interface is very close to the dike) and large values of $p_I d$ (corresponds to short wave region and the interface is far from the free surface) the wave transmission in SM due to interface becomes insignificant. On the other hand, for intermediate frequency range, the waves in SM transmitted significantly due to the presence of the interface and this may be the reason for smaller reflection coefficients in SM observed in the present study as compared to the wave reflection in the single-layer fluid. When the surface obstacle is above the interface, the general trend of the IM wave reflection observed in the two-layer

fluid is found to be similar to the one observed for a bottom obstacle case in a single-layer fluid (see Fig. 2 of Mei and Black (1969)). However, when the surface obstacle touches the interface or it extends beyond the interface (IPSO) the reflection in IM is found to be 100 %.

The variation of single dike reflection coefficients in SM and IM versus $p_I d$ are plotted in Fig. 4.4 (a) and (b) respectively for different values of H/d . In Fig. 4.4 (a), for all values of H/d , with an increase in $p_I d$, the wave reflection in SM increases and attains a 100 % reflection in the deep water region. In case of intermediate water depth, wave reflection in SM attains minimum for $H/d = 5$ and maximum for $H/d = 1.5$. On the other hand, for $d/H < h/H$, the general trend of reflection coefficient in IM follows an oscillating pattern and it attains a zero reflection in the deep water region (Fig. 4.4 (b)). This is due to the fact that in the deep water region, the dike is far from the interface and hence it has a negligible impact on waves in IM. However, as the dike approaches toward the interface, the wave reflection in IM increases sharply and attains a 100 % reflection over the entire frequency range in case of IPSO ($d \geq h$). This is because in such situation the propagation of internal waves through the interface as well as free surface is completely blocked by the rigid dike.

The effect of dike width to mean wetted draft ratio a/d on single dike reflection coefficients in SM and IM for the single dike are shown in Fig. 4.5 (a) and (b) respectively. It is observed that with an increase in a/d ratio, the reflection in both SM and IM increases. This is expected because an increase in dike width will enhance the wave reflection and when the width becomes infinitely large, the wave reflection in SM become 100 % over entire frequency range because in such situation there will not be any transmitted wave in SM. On the other hand the wave reflection in IM is similar to the one observed in the case of a bottom obstacle in a single-layer fluid (see Fig. 2 of Mei and Black (1969)).

The effect of depth ratio h/H on the single dike reflection coefficients in SM and IM for the single dike are shown in Fig. 4.6 (a) and (b) respectively. In Fig. 4.6 (a), it is observed that the reflection coefficients in SM for $h/H = 0.1$ and $h/H = 0.25$ are almost same except a little change in the intermediate frequency range. Similarly, except in the

shallow water region the difference in the values of wave reflection in SM is marginal for $h/H = 0.5$, $h/H = 0.75$ and $h/H = 0.9$. On the other hand, the reflection coefficient in IM is found to be increasing with a decrease in h/H ratio. It is obvious because as the interface approaches toward the bottom of the dike, the influence of dike on wave motion in IM is higher, which ultimately leads to a higher wave reflection. When the interface touches the bottom of the dike ($h/H = 0.25$) or it is above the dike bottom (the case of IPSO, $h/H = 0.1$), wave reflection in IM becomes 100 % and this observation is similar to the one observed in Fig. 4.4 (b).

The single dike reflection coefficients versus $p_I d$ are plotted in SM and IM for various values of s in Fig. 4.7 (a) and (b) respectively. In general it is observed that wave reflection in SM increases with an increase in the value of s and a reverse trend is observed in case of IM wave reflection.

In Fig. 4.8, results for the reflection coefficients in SM and IM versus $p_I d$ are plotted for different values of b/H in case of a pair of identical dikes. A comparison is made between the results obtained by the matched-eigenfunction-expansion method and the WSAM. It is observed that WSAM results for reflection coefficients match closely when the dikes are widely spaced. However, the agreement is closer in the case of wave reflection in IM. From both the methods in a narrow bandwidth of frequency, the reflection coefficients in SM reduce suddenly to a very small values. It is observed that with an increase in the gap between the dikes, more number of these narrow bandwidths of frequency corresponding to small wave reflection will appear.

4.5.2 Hydrodynamic Forces

It is very interesting to analyze the vertical and horizontal forces experienced by the surface-piercing dikes as in general they don't have a very strong bottom foundation. The definitions of horizontal force HF and vertical force VF are similar to the one explained in McIver (1986) and is given by:

$$HF = i\omega\rho\phi_0 \int_0^d (\phi(a, y) - \phi(-a, y))dy, \quad (4.56)$$

and

$$VF = i\omega\rho\phi_0 \int_{-a}^a \phi(x, d)dx. \quad (4.57)$$

It is clear that symmetric velocity potential does not contribute to the horizontal wave force. Similar definitions can be obtained for the horizontal and vertical forces, HF_n 's and VF_n 's respectively ($n = 1$ corresponds to the first dike which is exposed to the incident waves, and $n = 2$ corresponds to the second dike) in the case of a pair of identical dikes (see, McIver (1986)). The behavior of horizontal forces, HF_n 's and vertical forces, VF_n 's per unit incident wave amplitude and length of dikes is analyzed in the subsequent paragraphs.

The behavior of horizontal and vertical hydrodynamic forces experienced by a single dike is presented in Fig. 4.9 and 4.10. Resonance behavior is observed in the case of horizontal hydrodynamic force, which is similar to the one explained in McIver (1986). However, it is observed that the magnitude of resonating horizontal hydrodynamic force increases with a decrease in H/d value and an increase in a/d value. On the other hand, the vertical hydrodynamic force increases with an increase in $p_I a$ and in the deep water region the magnitude of vertical hydrodynamic force is high for small H/d and a/d values.

The variation of horizontal forces, HF_n 's and vertical forces, VF_n 's for $n = 1, 2$ in the case of a pair of identical dikes is studied for different b/H values in Fig. 4.11 and 4.12 respectively. Similar to McIver (1986) resonance behaviors are observed in both the cases of horizontal and vertical hydrodynamic forces. However, it is observed that the magnitude of resonating horizontal hydrodynamic force is higher on the second dike as compared to that on the first dike. On the other hand, the magnitudes of resonating vertical hydrodynamic forces on both the dikes are of same order. Moreover, in the case of vertical hydrodynamic force the number of resonating peaks are found to be high for smaller value of b/H , whilst the magnitude of vertical hydrodynamic force at resonance is found to be high for larger value of b/H .

4.5.3 Summary of Important Observations

The important observations from the present numerical results for surface-piercing dikes are summarized point wise as below:

1. Present two-layer fluid results for reflection coefficients in SM are found to be similar to that obtained for single-layer fluid by Mei and Black (1969).
2. When the surface obstacle is above the interface, the general trend of the IM wave reflection is found to be similar to the one obtained for a bottom obstacle case in a single-layer fluid by Mei and Black (1969).
3. When dike is placed nearer to the interface, the wave reflection in IM increases sharply and attains a 100 % reflection over the entire frequency range in case of IPSO ($d \geq h$).
4. With an increase in a/d ratio, the reflection in both SM and IM increases.
5. The reflection coefficient in IM increases with decrease in h/H ratio. When the interface touches the bottom of the dike or it is above the dike bottom wave reflection in IM becomes 100 %.
6. The wave reflection in SM increases with an increases in the value of s and a reverse trend is observed in case of IM wave reflection.
7. In case of a pair of identical dikes, WSAM results for wave reflection match closely with the results obtained by matched-eigenfunction-expansion method when the dikes are widely spaced. The agreement is closer in the case of wave reflection in IM.
8. In case of a pair of identical dikes, in a narrow bandwidth of frequency, the reflection coefficients in SM reduce suddenly to very small values. With an increase in the gap between the dikes more number of these narrow bandwidths of frequency corresponding to small wave reflection will appear.
9. Similar to McIver (1986) resonance behavior is observed in case of horizontal hydrodynamic forces experienced by a single dike and the magnitude of resonating horizontal hydrodynamic force increases with a decrease in H/d value and an increase in a/d value.

10. The vertical hydrodynamic force experienced by a single dike increases with an increase in $p_I a$ and in deep water region the magnitude of vertical hydrodynamic force is high for small H/d and a/d values.
11. Similar to McIver (1986) resonance behaviors are observed in both cases of horizontal and vertical hydrodynamic forces for a pair of identical dikes.
12. As compared to first dike the magnitude of resonating horizontal hydrodynamic force on the second dike is found to be higher.
13. The magnitude of resonating vertical hydrodynamic forces on both the dikes are of same order.
14. In case of vertical hydrodynamic force the number of resonating peaks are high for small b/H values, whilst the magnitude of vertical hydrodynamic force at resonance is found to be high for large values of b/H .

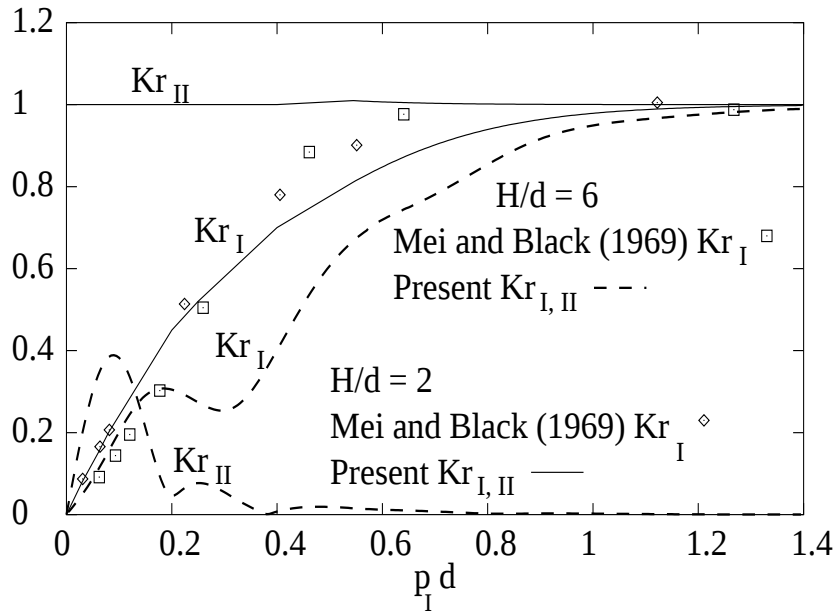
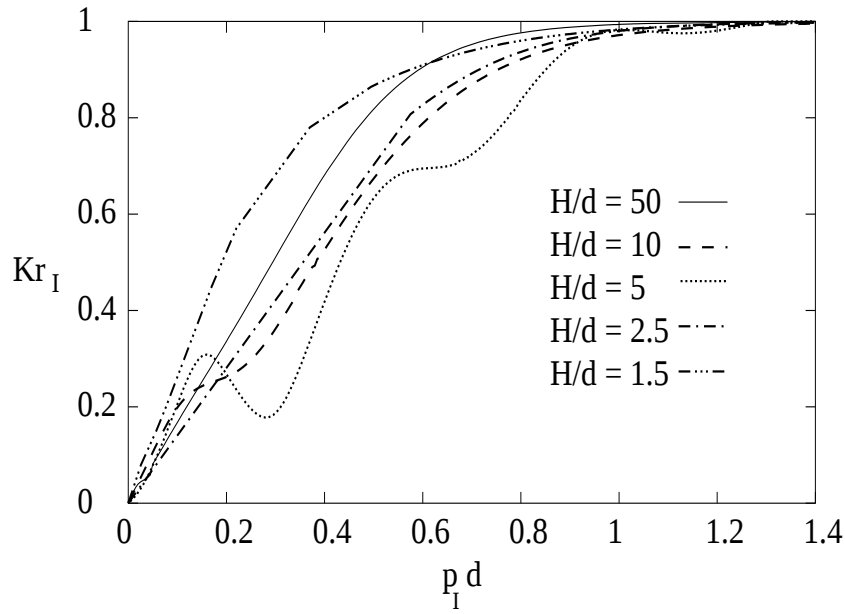
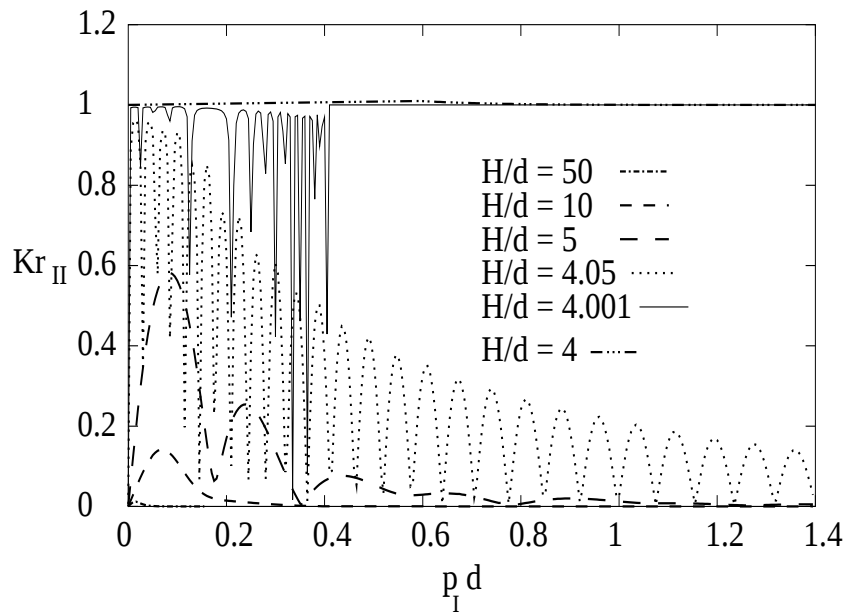


Figure 4.3: Comparison of reflection coefficients in SM, Kr_I and IM, Kr_{II} versus $p_I d$ for a single surface-piercing dike at different H/d values, $a/d = 1.0$, $h/H = 0.25$ and $s = 0.75$ with Mei and Black (1969).

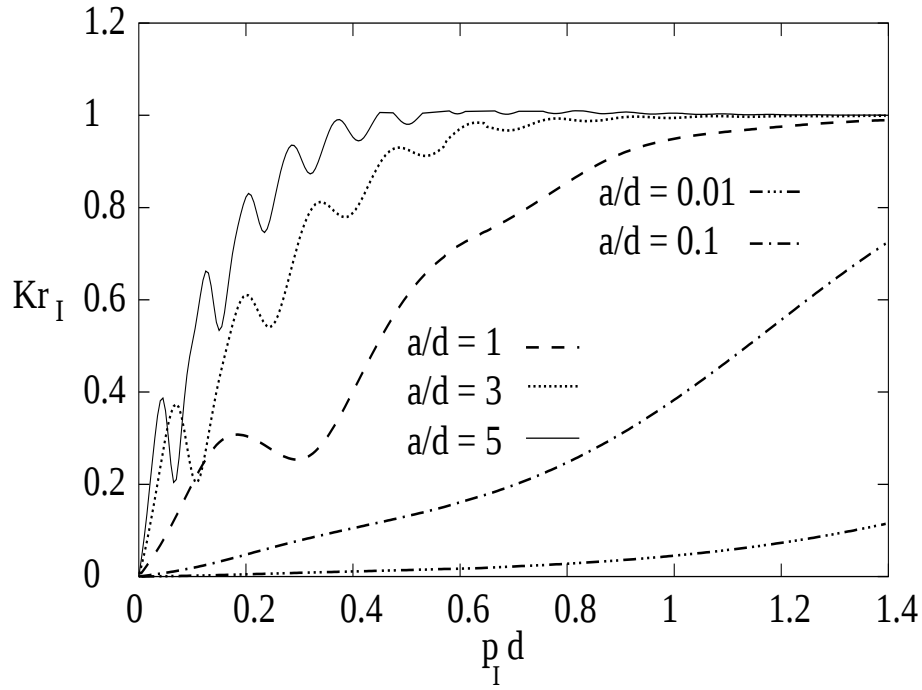


(a)

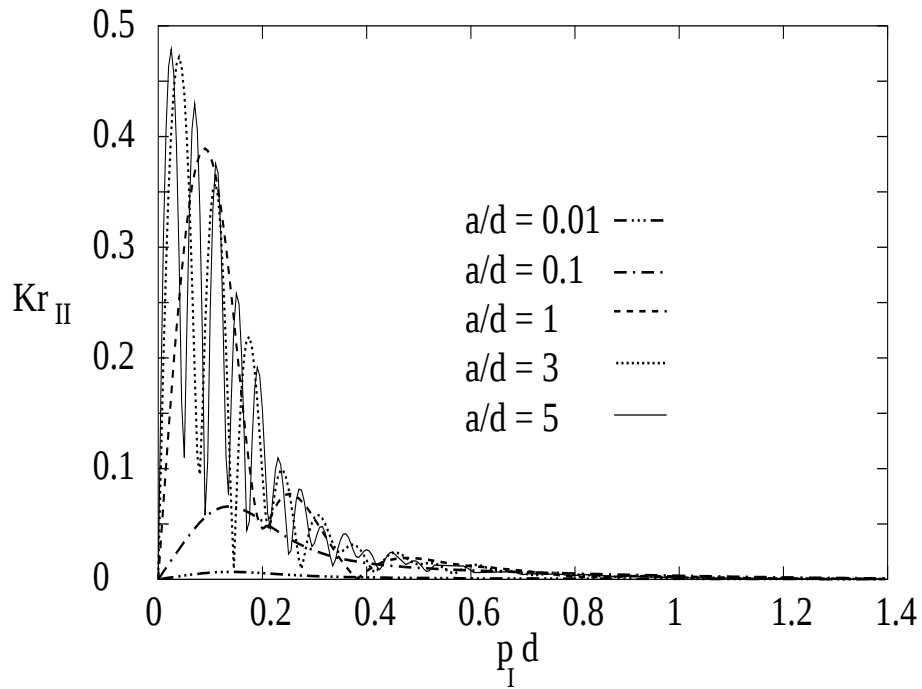


(b)

Figure 4.4: Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I d$ for a single surface-piercing dike at different H/d values, $a/d = 1.0$, $h/H = 0.25$ and $s = 0.75$.

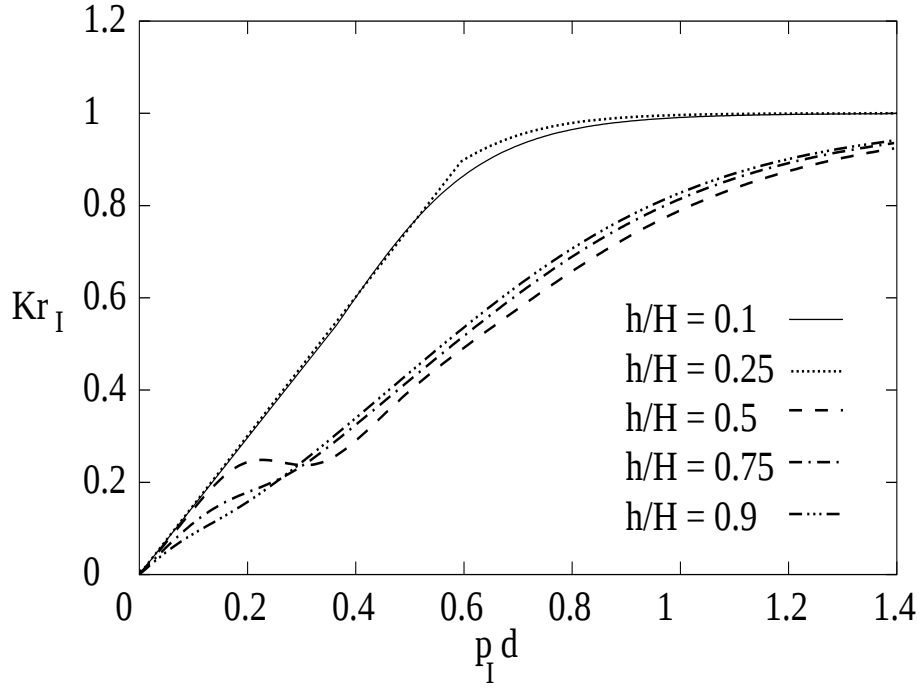


(a)

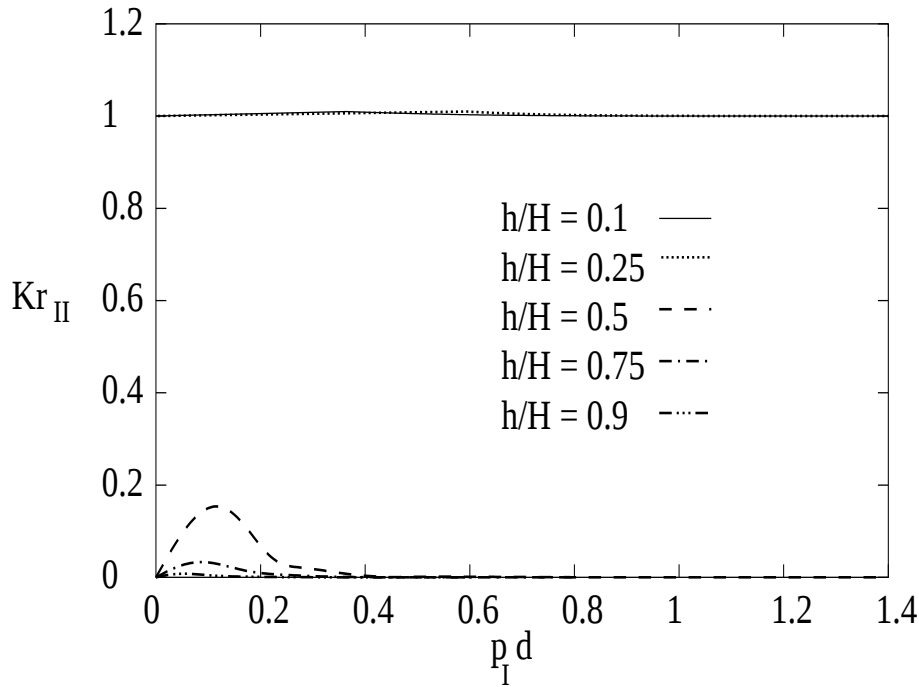


(b)

Figure 4.5: Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I d$ for a single surface-piercing dike at different a/d values, $H/d = 6.0$, $h/H = 0.25$ and $s = 0.75$.

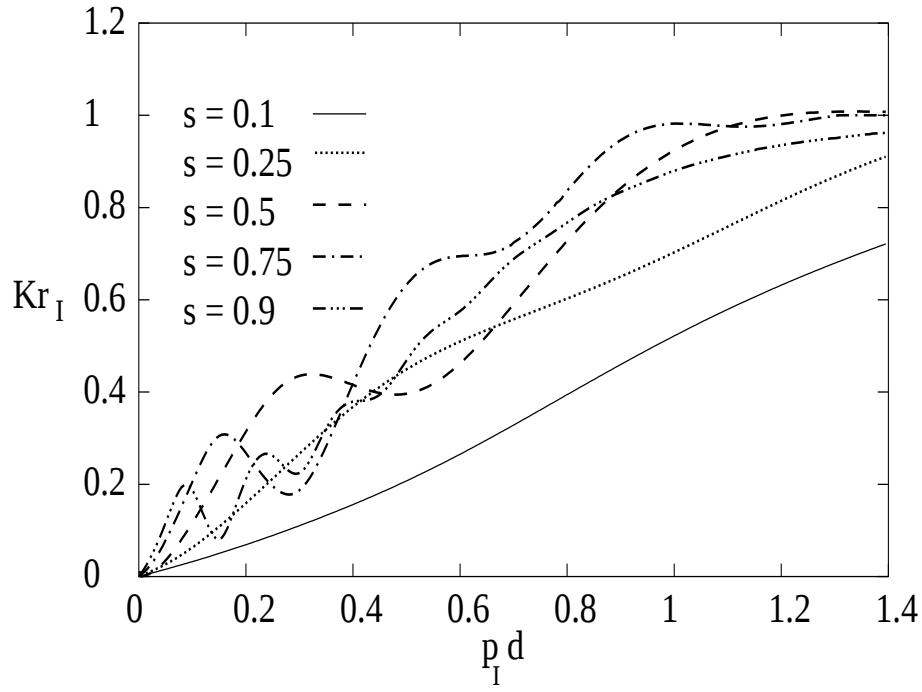


(a)

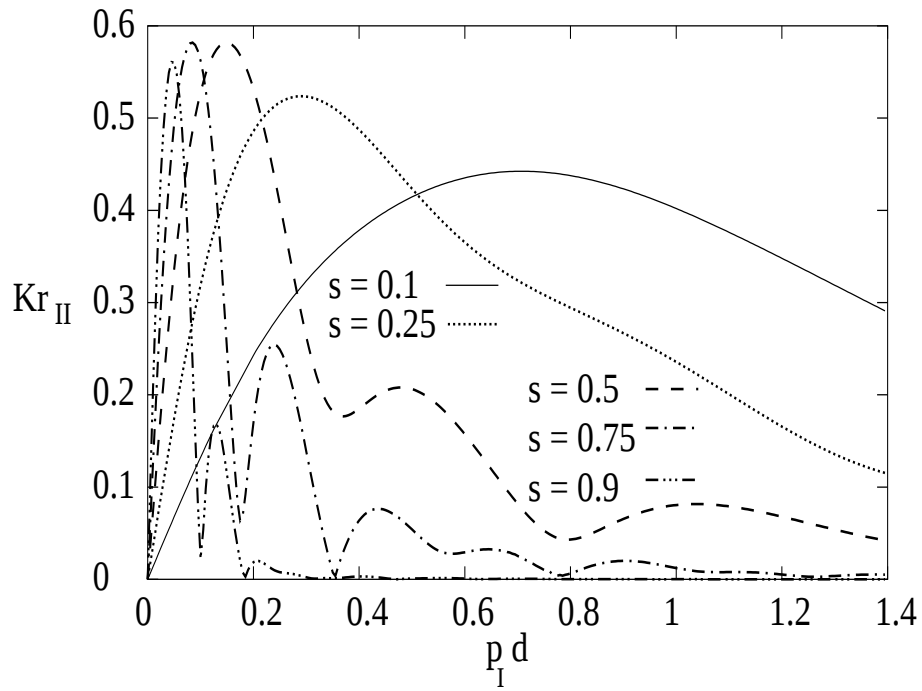


(b)

Figure 4.6: Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I d$ for a single surface-piercing dike at different h/H values, $H/d = 5.0$, $a/d = 1.0$ and $s = 0.75$.

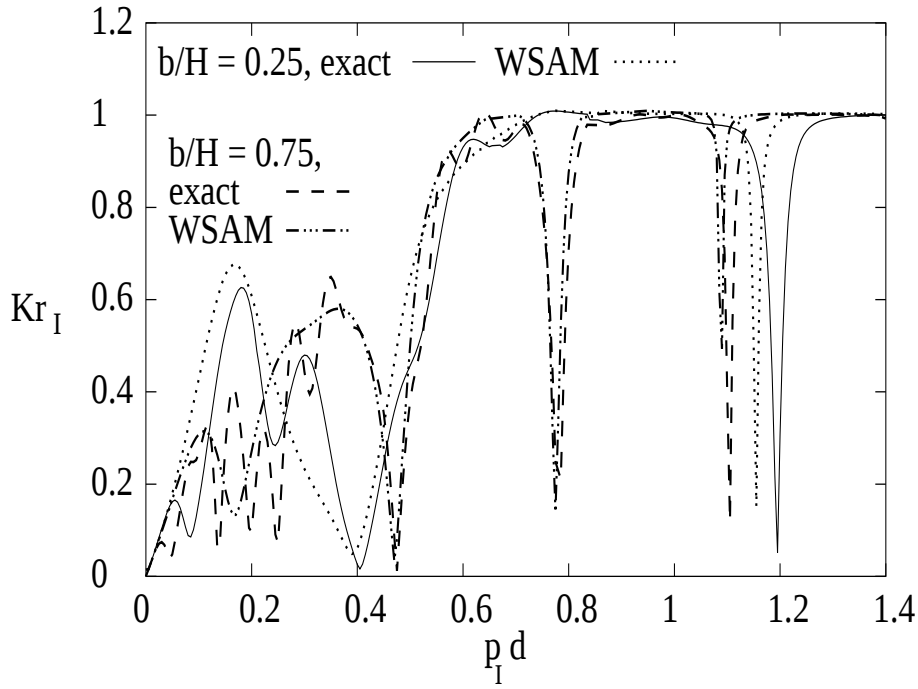


(a)

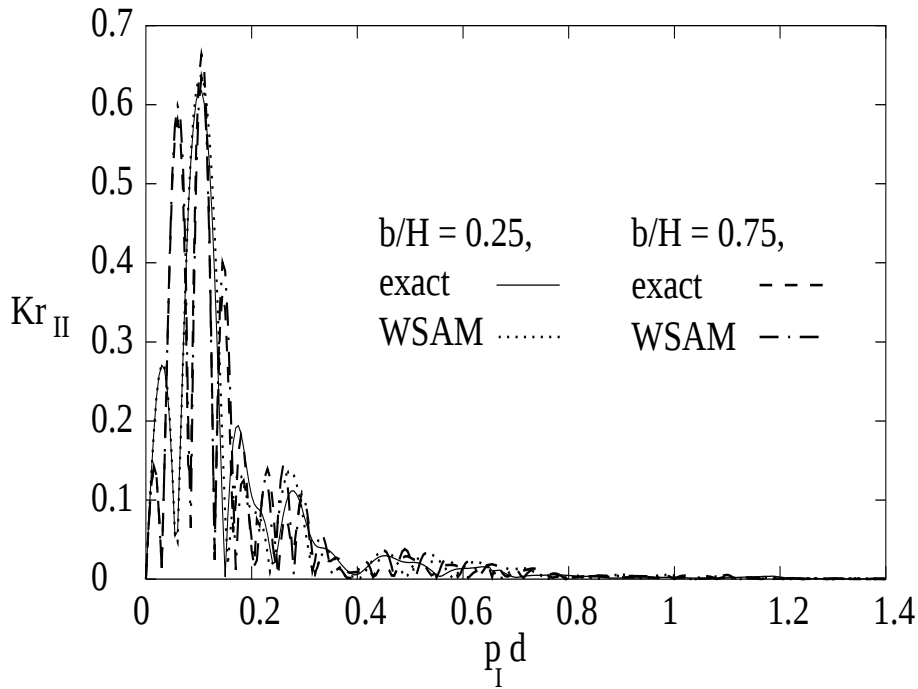


(b)

Figure 4.7: Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I d$ for a single surface-piercing dike at different s values, $H/d = 5.0$, $a/d = 1.0$ and $h/H = 0.25$.

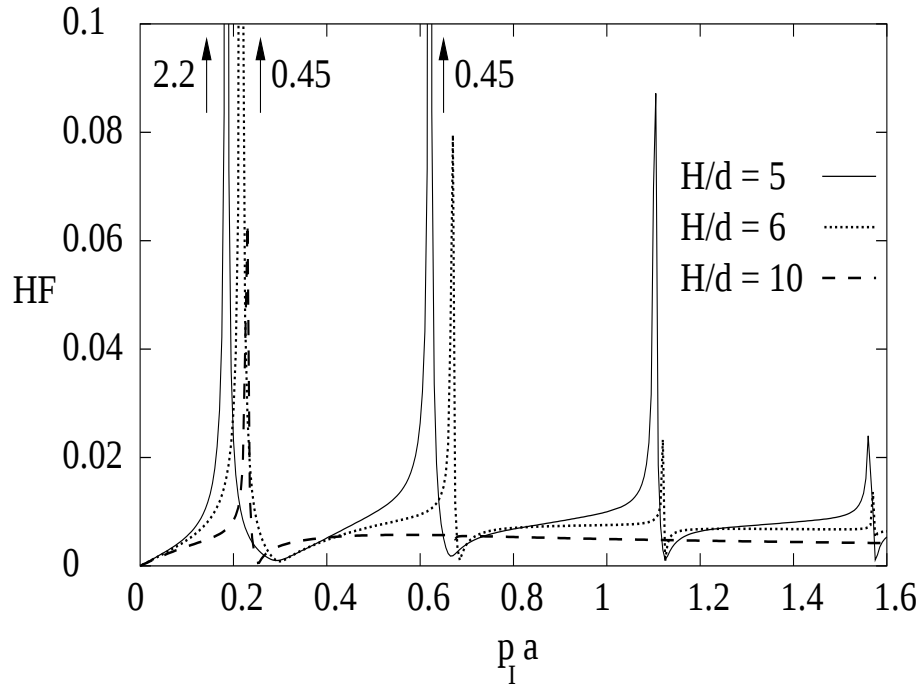


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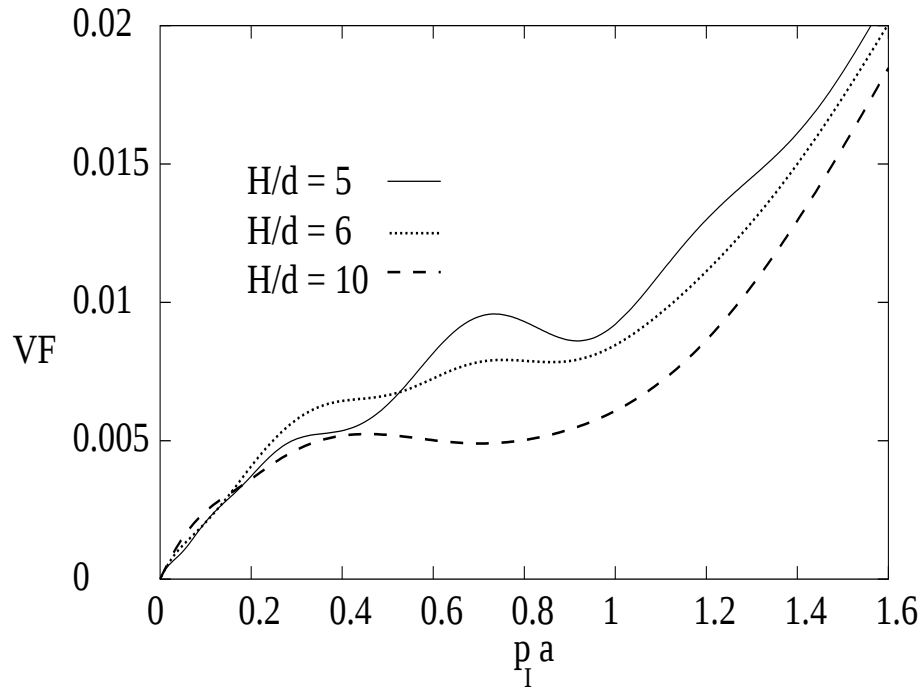


(b)

Figure 4.8: Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I d$ for a pair of identical surface-piercing dikes at different b/H values, $H/d = 6.0$, $a/d = 1.0$, $s = 0.75$ and $h/H = 0.25$.

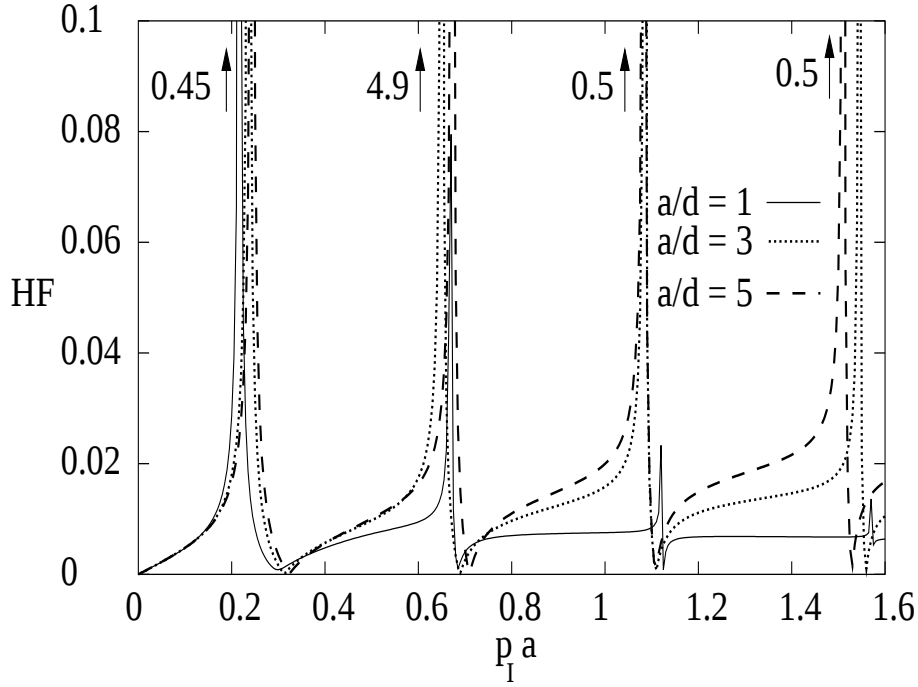


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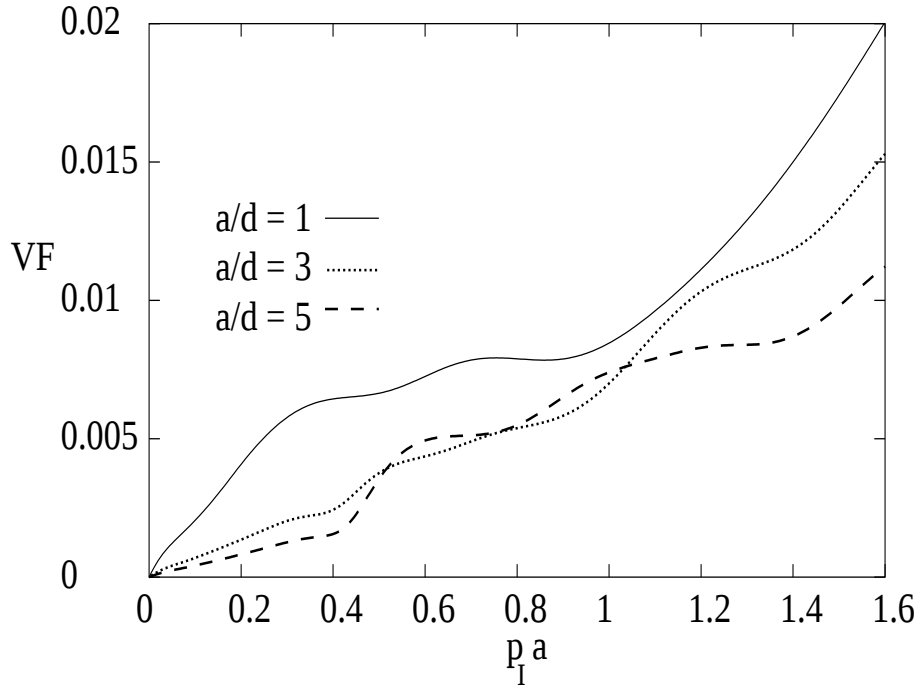


(b)

Figure 4.9: (a) Horizontal force, HF and (b) Vertical force, VF per unit incident wave amplitude and length of dike in MN/m^2 for a single surface-piercing dike at different H/d values, $a/d = 1.0$, $s = 0.75$ and $h/H = 0.25$.

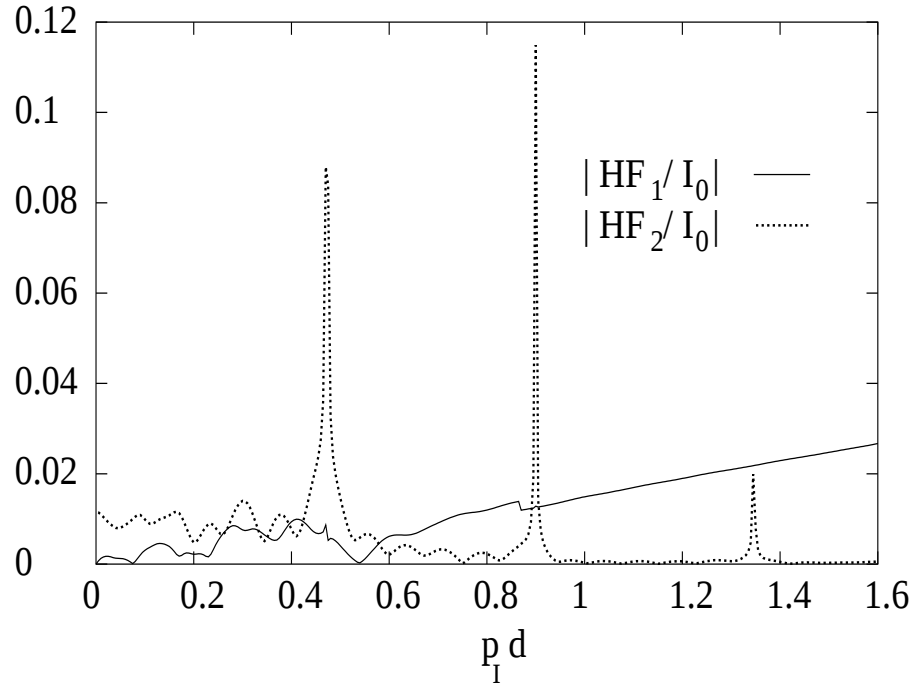


(a)

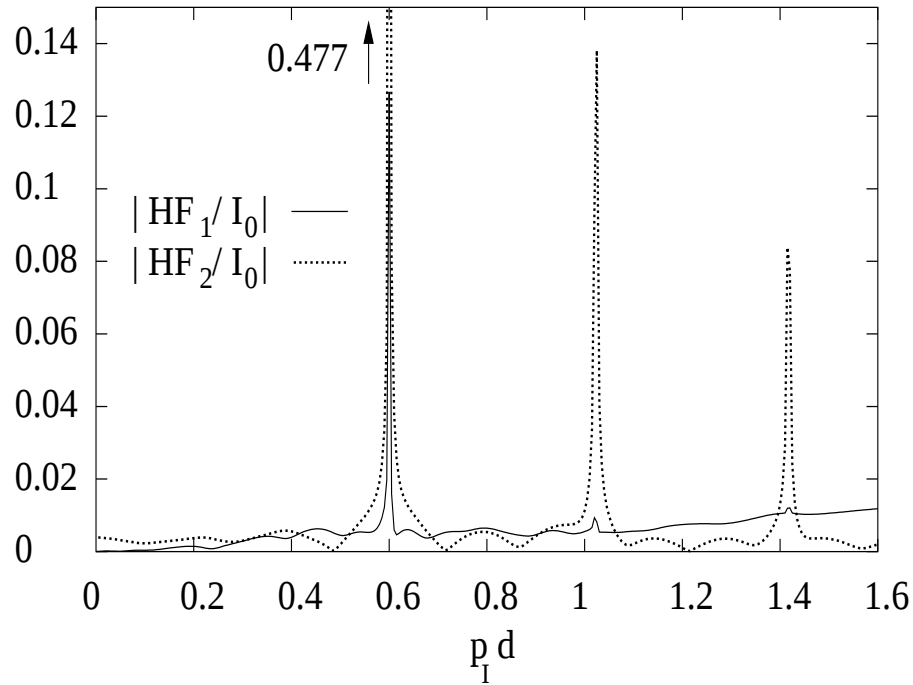


(b)

Figure 4.10: (a) Horizontal force, HF and (b) Vertical force, VF per unit incident wave amplitude and length of dike in MN/m^2 for a single surface-piercing dike at different a/d values, $H/d = 6.0$, $s = 0.75$ and $h/H = 0.25$.

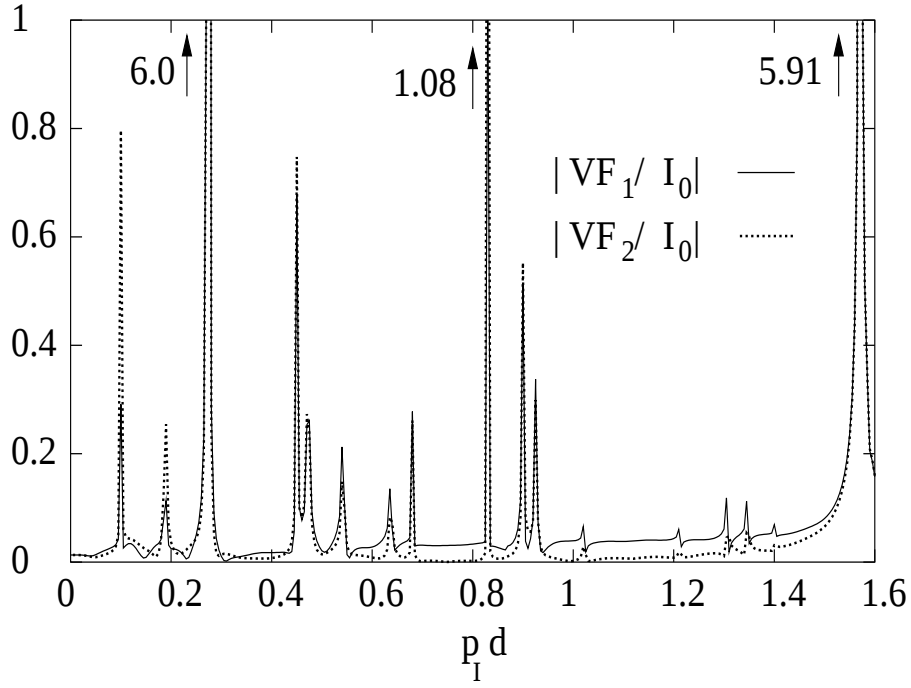


(a)

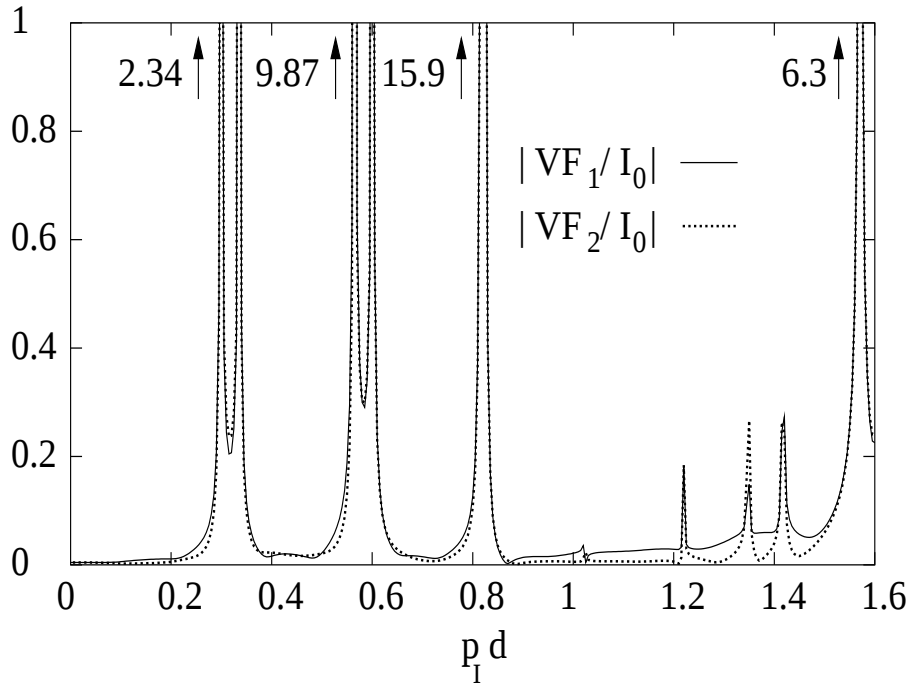


(b)

Figure 4.11: Horizontal force on first, $|HF_1/I_0|$ and second, $|HF_2/I_0|$ dike in MN/m^2 for (a) $b/H = 0.25$ (b) $b/H = 0.75$, at $H/d = 6.0$, $a/d = 0.1$, $s = 0.75$ and $h/H = 0.25$.



(a)



(b)

Figure 4.12: Vertical force on first, $|VF_1/I_0|$ and second, $|VF_2/I_0|$ dike in MN/m² for (a) $b/H = 0.25$ (b) $b/H = 0.75$, at $H/d = 6.0$, $a/d = 0.1$, $s = 0.75$ and $h/H = 0.25$.

Chapter 5

WAVE SCATTERING BY BOTTOM-STANDING DIKES

5.1 INTRODUCTION

Bottom-standing dikes are considered in this chapter. Like the case of surface-piercing dikes discussed in Chapter 4, in the present chapter also both the cases of a single and a pair of identical bottom-standing dikes are considered within the context of linearized-theory of water waves. Moreover, dikes are again assumed as rigid structures for fluid flow.

The mathematical model for bottom-standing dikes is similar to that considered in the case of surface-piercing dikes. The major difference is that there is no dike covered region in the case of bottom standing dikes. The Cartesian co-ordinate system is again chosen with x -axis in the direction of wave propagation and y -axis in the downward direction. Similarly, dikes are approximated as cylinders of rectangular geometry and are placed in a two-layer fluid of finite depth. The rectangular dikes are completely immersed in the two-layer fluid. The problem is considered in the two-dimensions under the assumptions that the dikes have parallel generators and are of sufficient length for normal waves incidence. In the two-layer fluid, the upper fluid has a free surface (undisturbed free surface located at $y = 0$) and the two fluids are separated by a common interface (undisturbed interface

located at $y = h$), each fluid is of infinite horizontal extent occupying the region $-\infty < x < \infty$; $0 < y < h$ in case of the upper fluid of density ρ_1 , and $-\infty < x < \infty$; $h < y < H$ in case of the lower fluid of density ρ_2 . The flow is assumed to be irrotational and simple harmonic in time with angular frequency ω . Hence as defined earlier the velocity potential $\Phi(x, y, t)$ exists such that $\Phi(x, y, t) = \text{Re}[\phi(x, y)\exp(-i\omega t)]$.

5.2 MODEL IN THE CASE OF A SINGLE DIKE

5.2.1 Definition of the Physical Problem

In the present subsection, solution of wave scattering by a single bottom-standing dike is analyzed in a two-layer fluid which is further generalized to study the wave scattering by a pair of identical bottom-standing dikes in the subsequent section. Fig. 5.1 illustrate the problem under consideration. The dike is of width $2a$ and draft d . The origin $(0, 0)$ is chosen to be the point, where the mean free surface intersect with the vertical axis passing through the center of the dike.

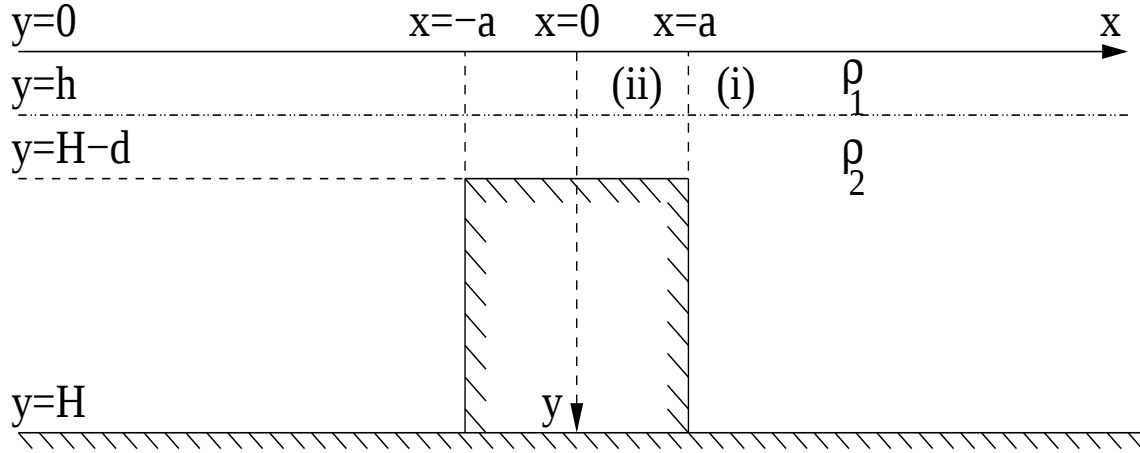


Figure 5.1: Definition sketch for single bottom-standing dike.

5.2.2 Velocity Potentials

Following a similar procedure as in the case of surface-piercing dike, in the present case also the solution is written as the sum of a symmetric and an anti-symmetric parts (see

Eq. 4.1). The line of symmetry is the line $x = 0$, which passes through the center of the dike (see Fig. 5.1). With this decomposition of velocity potential, the boundary value problem reduces to a simpler problem in the region $x \geq 0$ only. The eigenfunction-expansion method similar to the one used in the case of a single surface-piercing dike is adopted in the present problem also to construct the velocity potentials. The symmetric and anti-symmetric parts of the velocity potential in the region (i) are same as that described in case of a surface-piercing dike (see Eqs. 4.4 and 4.5). The eigenvalues p_n 's in the region (i) satisfy the same dispersion relation in p as given in Eq. 4.8. The symmetric and anti-symmetric parts of the velocity potential in region shallow (ii) are as given below.

$$\phi_2^s = \sum_{n=I,II,1}^{\infty} B_n^s \frac{\cos q_n x}{\cos q_n a} \chi_n(y), \quad (5.1)$$

and

$$\phi_2^a = \sum_{n=I,II,1}^{\infty} B_n^a \frac{\sin q_n x}{\sin q_n a} \chi_n(y). \quad (5.2)$$

The vertical eigenfunctions $\chi_n(y)$'s, satisfying governing equation (Eq. 3.1) along with the conditions Eqs. 3.12 – 3.14 and Eq. 3.15 at the top of the dike ($y = H - d$) are given by

$$\chi_n(y) = \begin{cases} \frac{L_n^{-1} \sinh q_n (H - d - h) [q_n \cosh q_n y - K \sinh q_n y]}{K \cosh q_n h - q_n \sinh q_n h}, & \text{for } 0 < y < h, \\ L_n^{-1} \cosh q_n (H - d - y), & \text{for } h < y < (H - d), \end{cases} \quad (n = I, II, 1, 2, 3, \dots) \quad (5.3)$$

where

$$\begin{aligned} L_n^2 = & (4q_n)^{-1} (K \cosh q_n h - q_n \sinh q_n h)^{-2} [(K \cosh q_n h - q_n \sinh q_n h)^2 \\ & (2(H - d - h)q_n + \sinh 2q_n(H - d - h)) - s \sinh^2 q_n(H - d - h) \\ & (K^2(2q_n h - \sinh 2q_n h) - q_n^2(2q_n h + \sinh 2q_n h) + 2Kq_n(\cosh 2q_n h - 1))]. \end{aligned} \quad (5.4)$$

Similar to Eq. 4.8 the eigenvalues q_n 's satisfy the dispersion relation in q as given by

$$\begin{aligned} (1 - s)q^2 \tanh q(H - d - h) \tanh qh - qK[\tanh qh + \tanh q(H - d - h)] \\ + K^2[s \tanh q(H - d - h) \tanh qh + 1] = 0. \end{aligned} \quad (5.5)$$

It may be noted that in region (ii), in case of $(H - d) > h$, the vertical eigenfunctions have two propagating modes because of the presence of the free surface and the interface. The vertical eigenfunctions in region (ii) are similar with the vertical eigenfunctions in the region (i). On the other hand, in case of $(H - d) \leq h$ (special case of bottom obstacle), the vertical eigenfunctions χ_n 's do not satisfy the interface conditions Eqs. (3.13 and 3.14) and in such situation like a single-layer fluid, only one propagative mode exists because of the free surface. Hence, similar to single-layer fluid, the term $n = II$ does not appear in the expression of eigenfunctions and the corresponding vertical eigenfunctions becomes

$$\chi_n = L_n^{-1} \frac{\cosh q_n(H - d - y)}{\cosh q_n(H - d)}, \quad (n = I, 1, 2, \dots), \quad (5.6)$$

where

$$L_n^2 = \frac{2q_n(H - d) + \sinh 2q_n(H - d)}{4 q_n \cosh^2 q_n(H - d)}, \quad (5.7)$$

and q_n satisfy the relation

$$K = q_n \tanh q_n(H - d), \text{ for } n = I, 1, 2, \dots \quad (5.8)$$

The radiation condition, definition of reflection and transmission coefficients, condition on the wetted draft of the dike, continuity condition across the gap are similar to the case of a single surface-piercing dike.

The eigenfunctions f_m 's are integrable in $0 < y < H$ having a single discontinuity at $y = h$, and are orthonormal with respect to the inner product as given in Eq. 4.26 for the case of surface-piercing dikes.

Similar to f_m 's, χ_n 's are also orthonormal with respect to the inner product

$$\langle \chi_n, \chi_m \rangle_3 = \begin{cases} s \int_0^h \chi_n \chi_m dy + \int_h^{H-d} \chi_n \chi_m dy, & \text{for } (H - d) > h, \\ \int_0^{H-d} \chi_n \chi_m dy, & \text{for } (H - d) \leq h. \end{cases} \quad (5.9)$$

5.2.3 General Solution Procedure

The matching procedure at $x = a$ for symmetric velocity potentials in the case of bottom-standing dike is very similar to that used in the case of surface-piercing dike. The resulting

equation is given by

$$ip_m A_m^s + \sum_{n=I,II,1}^{\infty} \alpha_{mn}^s A_n^s = \frac{1}{2} \sum_{r=I}^{II} (ip_r \delta_{mr} - \alpha_{mr}^s), \quad m = I, II, 1, \dots \quad (5.10)$$

The matching procedure at $x = a$ for anti-symmetric velocity potentials in the case of bottom-standing dike is also very similar to that used in the case of surface-piercing dike. The resulting equation is given by

$$ip_m A_m^a - \sum_{n=I,II,1}^{\infty} \alpha_{mn}^a A_n^a = \frac{1}{2} \sum_{r=I}^{II} (ip_r \delta_{mr} + \alpha_{mr}^a), \quad m = I, II, 1, \dots, \quad (5.11)$$

where the expressions for $\alpha_{mn}^{s,a}$ are same as defined in Eqs. (4.31 and 4.33).

The systems of equations (5.10) and (5.11) are solved using Gauss-elimination method to obtain the various physical quantities of interest. Number of evanescent modes in the series are selected based on the experience of the numerical convergence experiment.

5.3 MODEL IN THE CASE OF A PAIR OF IDENTICAL DIKES

5.3.1 Definition of the Physical Problem

In the present section, solution for scattering of incident wave trains by a pair of identical bottom-standing dikes in a two-layer fluid is considered. The wave scattering by a pair of identical bottom-standing dikes, as shown in Fig. 5.2 is solved. Dikes are again of width $2a$ and draft d . The origin $(0,0)$ is chosen to be at the center of dike located on the right-hand of Fig. 5.2.

5.3.2 Velocity Potentials

The symmetry of the problem is again exploited by writing the solution as the sum of a symmetric and an anti-symmetric parts. The line of symmetry for present problem is $x = -(a + b)$ (see Fig. 5.2). Hence similar to single surface-piercing dike we can write

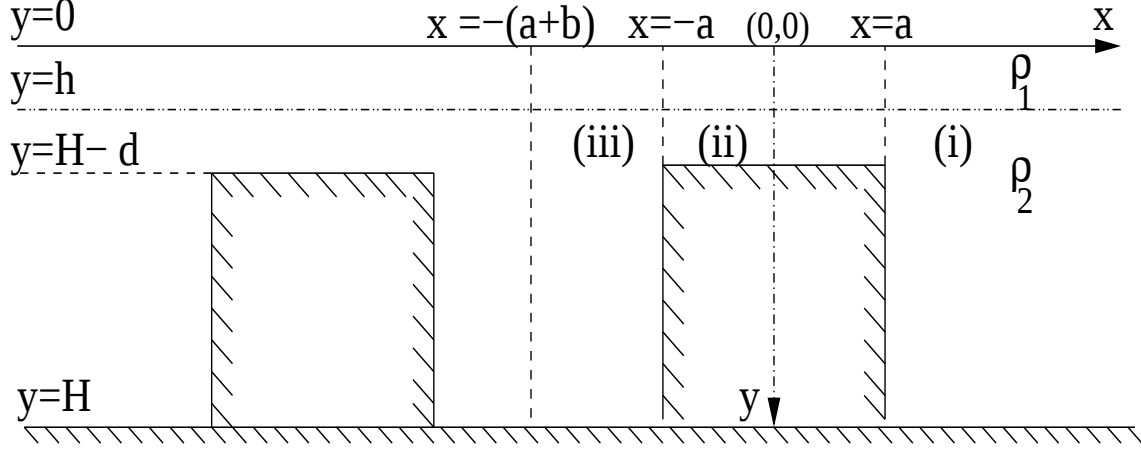


Figure 5.2: Definition sketch for a pair of identical bottom-standing dikes.

velocity potential $\phi(x, y)$ as a sum of symmetric and anti-symmetric parts as in Eq. (4.1). With this decomposition of velocity potential, the boundary value problem reduces to a simpler problem in the region $x \geq -(a+b)$ only. The eigenfunction-expansion method similar to the one used in the case of a single surface-piercing dike is adopted in the present problem also to construct the velocity potentials.

Similar to the case of a pair of identical surface-piercing dikes there are now two regions which are beyond the dike width (region (i) and (iii)) and two matching boundaries. The symmetric and anti-symmetric parts of the velocity potential in the region (i) are same as that presented in case of a pair of identical surface-piercing dikes (see Eqs. 4.35 and 4.37). The symmetric part of the velocity potential in region (iii) is as given below.

$$\phi_3^s = \sum_{n=I,II,1}^{\infty} D_n^s \frac{\cos p_n(x+a+b)}{\sin p_nb} f_n(y). \quad (5.12)$$

The anti-symmetric part of the velocity potential in region (iii) is as given below.

$$\phi_3^a = \sum_{n=I,II,1}^{\infty} D_n^a \frac{\sin p_n(x+a+b)}{\cos p_nb} f_n(y). \quad (5.13)$$

The symmetric part of the velocity potential in shallow region (ii) is as given below.

$$\phi_2^s = \sum_{n=I,II,1}^{\infty} \left[B_n^s \frac{\cos q_n x}{\cos q_na} + C_n^s \frac{\sin q_n x}{\sin q_na} \right] \chi_n(y). \quad (5.14)$$

The anti-symmetric part of the velocity potential in region (ii) is given below.

$$\phi_2^a = \sum_{n=I,II,1}^{\infty} \left[B_n^a \frac{\cos q_n x}{\cos q_na} + C_n^a \frac{\sin q_n x}{\sin q_na} \right] \chi_n(y). \quad (5.15)$$

The vertical eigenfunctions and dispersion relations in regions beyond the dike width and in region over the dikes are same as described in the case of a single bottom-standing dike. The radiation condition, definition of reflection and transmission coefficients, condition on the wetted draft of the dike, continuity condition across the gap are also similar to the case of a single surface-piercing dike. Moreover, the orthonormality conditions as described in the case of a single bottom-standing dike is also applicable in the present problem.

5.3.3 General Solution Procedure

Matching procedure for $x = a$ is very similar to that used for the single bottom-standing dike case. The resulting equations for the unknown coefficients are

$$p_m(iA_m^s - D_m^s) - \sum_{n=I, II, 1}^{\infty} \alpha_{mn}^a (A_n^s - D_n^s \cot p_n b) = \frac{1}{2} \sum_{r=I}^{II} (ip_r \delta_{mr} + \alpha_{mr}^a),$$

$$m = I, II, 1, \dots, \quad (5.16)$$

and

$$p_m(iA_m^s + D_m^s) + \sum_{n=I, II, 1}^{\infty} \alpha_{mn}^s (A_n^s + D_n^s \cot p_n b) = \frac{1}{2} \sum_{r=I}^{II} (ip_r \delta_{mr} - \alpha_{mr}^s),$$

$$m = I, II, 1, \dots, \quad (5.17)$$

where $\alpha_{mn}^{s,a}$ are the same as defined in Eqs. (4.31 and 4.33). The remaining coefficients, for the velocity potential ϕ_2^s (Eq. 5.14), are given by

$$2B_m^s = \frac{1}{2} \sum_{n=I}^{II} C_{nm} + \sum_{n=I, II, 1}^{\infty} C_{nm} (A_n^s + D_n^s \cot p_n b), \quad m = I, II, 1, \dots, \quad (5.18)$$

and

$$2C_m^s = \frac{1}{2} \sum_{n=I}^{II} C_{nm} + \sum_{n=I, II, 1}^{\infty} C_{nm} (A_n^s - D_n^s \cot p_n b), \quad m = I, II, 1, \dots. \quad (5.19)$$

where $C_{nm} = \langle f_n, \chi_m \rangle$.

The final equations for the anti-symmetric velocity potential coefficients $\{A_n^a, D_n^a; n = I, II, 1, \dots\}$ and $\{B_n^s, C_n^a; n = 0, I, 1, \dots\}$ are similar in form to Eqs. 5.16 – 5.19

except that $\cot p_nb$ must be replace by $\tan p_nb$ throughout. This gives

$$p_m(iA_m^a - D_m^a) - \sum_{n=I, II, 1}^{\infty} \alpha_{mn}^a (A_n^a - D_n^a \tan p_nb) = \frac{1}{2} \sum_{r=I}^{II} (ip_r \delta_{mr} + \alpha_{mr}^a),$$

$$m = I, II, 1, \dots, \quad (5.20)$$

and

$$p_m(iA_m^a + D_m^a) + \sum_{n=I, II, 1}^{\infty} \alpha_{mn}^s (A_n^a + D_n^a \tan p_nb) = \frac{1}{2} \sum_{r=I}^{II} (ip_r \delta_{mr} - \alpha_{mr}^s),$$

$$m = I, II, 1, \dots. \quad (5.21)$$

The remaining coefficients, for the velocity potential ϕ_2^a (Eq. 5.15), are given by

$$2B_m^a = \frac{1}{2} \sum_{n=I}^{II} C_{nm} + \sum_{n=I, II, 1}^{\infty} C_{nm} (A_n^a + D_n^a \tan p_nb), \quad m = I, II, 1, \dots, \quad (5.22)$$

and

$$2C_m^a = \frac{1}{2} \sum_{n=I}^{II} C_{nm} + \sum_{n=I, II, 1}^{\infty} C_{nm} (A_n^a - D_n^a \tan p_nb), \quad m = I, II, 1, \dots. \quad (5.23)$$

The equation sets (5.16) and (5.17) are solved simultaneously for unknown coefficients $\{A_n^s, D_n^s, n = I, II, 1, \dots\}$. The equation sets (5.20) and (5.21) are solved simultaneously to obtain the unknowns $\{A_n^a, D_n^a, n = I, II, 1, \dots\}$. Again Gauss-elimination method is used to solve the matrix systems and evanescent modes in the series are selected based on the experience of the numerical convergence experiment.

All the equations (Eqs. 4.49 – 4.55) derived applying WSAM for the case of a pair of identical surface-piercing dikes in section (4.5) is also valid for the present case where a pair of bottom-standing dikes are considered in a two-layer fluid.

5.4 NUMERICAL RESULTS AND DISCUSSION

Numerical results are computed and analyzed for surface- and internal-wave scattering by a single and a pair of identical bottom-standing dikes in a two-layer fluid. The effects of various non-dimensional physical parameters on wave reflection in both SM and IM are analyzed. For convenience, the wave parameters are given in terms of the non-dimensional wave number p_Id , gap between the dikes p_lb , depth ratio h/H , fluid density ratio s along with the non-dimensional dike parameters given by a/d , H/d and b/H .

5.4.1 Reflected Energy

The variation of reflection coefficients in SM and IM versus $p_I(H - d)$ are plotted in Fig. 5.3 (a) and (b) respectively for different values of H/d . In general, it is observed that the wave reflection in both SM and IM are found to be increasing with a decrease in H/d . When the bottom-standing dike is in the lower fluid domain ($H - d > h$, i.e $H/d = 2.0$ and 3.0), with an increase in $p_I(H - d)$, the wave reflection in both SM and IM decreases and approaches to zero in the deep water region. This is because the bottom-standing dike has a negligible impact on the wave motion in both the modes in the deep water region. However, when the dike is extended up to the upper fluid (in case of IPBO, $H - d < h$, i.e $H/d = 1.05$ and 1.15), the wave reflection in SM is significantly high and attains a maximum value in the intermediate frequency range (Fig. 5.3 (a)). For IPBO, the wave reflection in IM is found to be increasing with an increase in $p_I(H - d)$ and the trend suggests that the reflection in IM attains 100 % reflection in the deep water region (Fig. 5.3 (b)). This is because in such situation the interface is far from the free surface and the waves in IM cannot get transmitted by the free surface and this will lead to a situation where there will be no wave transmission in IM.

The effect of dike width to mean wetted draft ratio a/d on reflection coefficients in SM and IM are shown in Fig. 5.4 (a) and (b) respectively. It is observed that with an increase in a/d ratio, the reflection in both SM and IM increases. The general trend of the reflection coefficients in both SM and IM is similar to the one observed in the case of a bottom-standing dike in a single-layer fluid (see Fig. 2 of Mei and Black (1969)).

The effect of interface location h/H on reflection coefficients in SM and IM are shown in Fig. 5.5 (a) and (b) respectively. In general it is observed that with a decrease in h/H ratio the reflection in SM increases (Fig. 5.5 (a)). On the other hand, the wave reflection in IM increases with an increase in the value of h/H (Fig. 5.5 (b)). However, the general trend of IPBO case reflection coefficients in both SM and IM is different than that of non-IPBO case.

Reflection coefficients are plotted versus $p_I(H - d)$ in SM and IM for various values of s in Fig. 5.6 (a) and (b) respectively. It is observed that the wave reflection in SM has

higher reflection peaks for higher values of s and a reverse trend is observed in case of IM wave reflection.

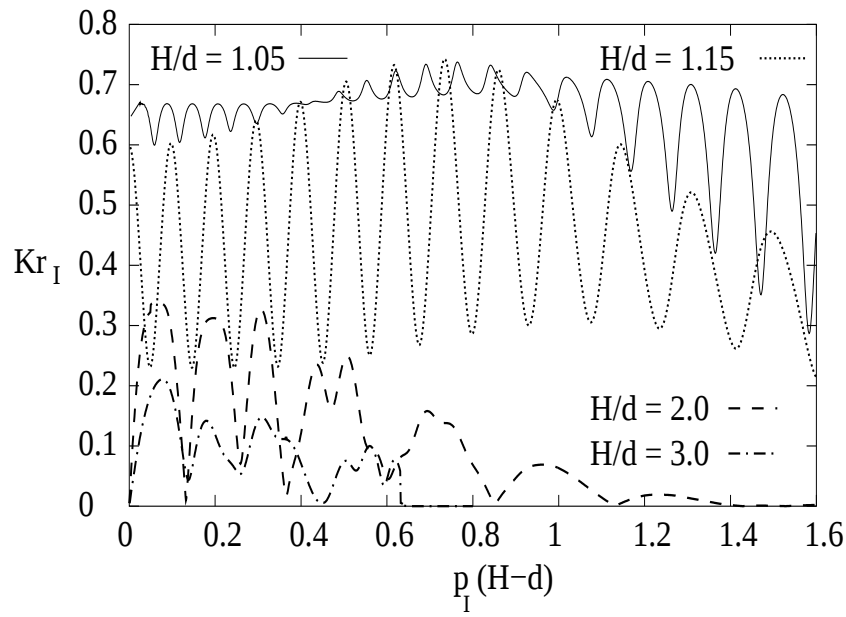
In Fig. 5.7 (a) and (b), results for the reflection coefficients in SM and IM versus $p_I d$ are plotted for different values of b/H in case of a pair of identical dikes. A comparison is made between the results obtained by the matched-eigenfunction-expansion method and the WSAM. Similar to case of pair of surface-piercing dikes, it is observed that WSAM results for reflection coefficients match closely when the dikes are widely spaced.

5.4.2 Summary of Important Observations

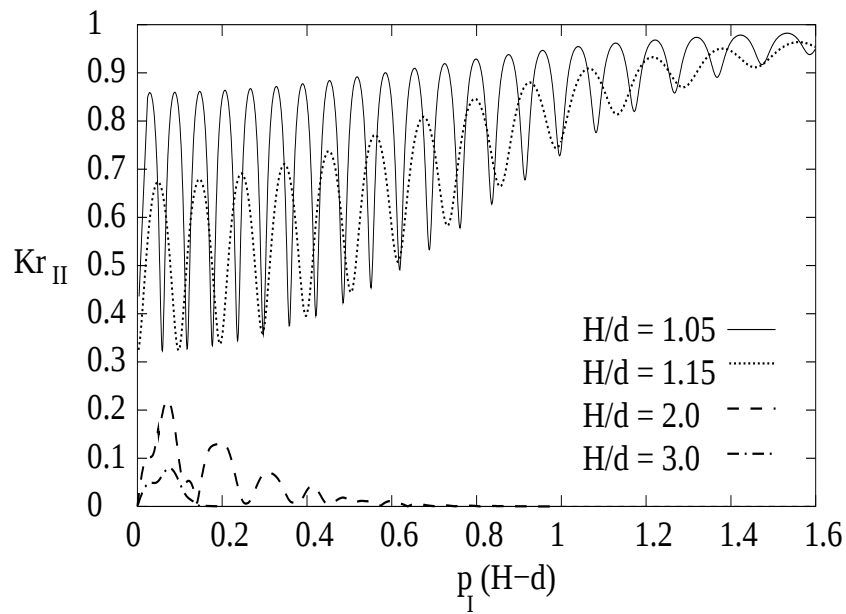
The important observations from the present numerical results for bottom-standing dikes are summarized point wise as below:

1. Wave reflection in both SM and IM are increasing with a decrease in H/d .
2. When the bottom-standing dike is in the lower fluid domain ($H - d > h$), with an increase in $p_I(H - d)$, the wave reflection in both SM and IM decreases and approaches to zero in the deep water region.
3. When the dike is extended up to the upper fluid (IPBO, $H - d < h$), the wave reflection in SM is significantly high and attains a maximum value in the intermediate frequency range.
4. For IPBO, the wave reflection in IM is increasing with an increase in $p_I(H - d)$ and the trend suggests that the reflection in IM attains 100 % reflection in the deep water region.
5. With increase in a/d ratio, the reflection in both SM and IM increases. The general trend of the reflection coefficients in both SM and IM is similar to the one observed in the case of a bottom-standing dike in a single-layer fluid (Fig. 2 of Mei and Black (1969)).
6. With a decrease in h/H ratio the wave reflection in SM increases and the wave reflection in IM decrease.
7. Wave reflection in SM has higher reflection peaks for higher values of s and a reverse trend is observed in case of IM wave reflection.

8. Similar to case of pair of surface-piercing dikes, reflection coefficients obtained by WSAM and matched-eigenfunction-expansion method for bottom-standing dikes match closely when the dikes are widely spaced.

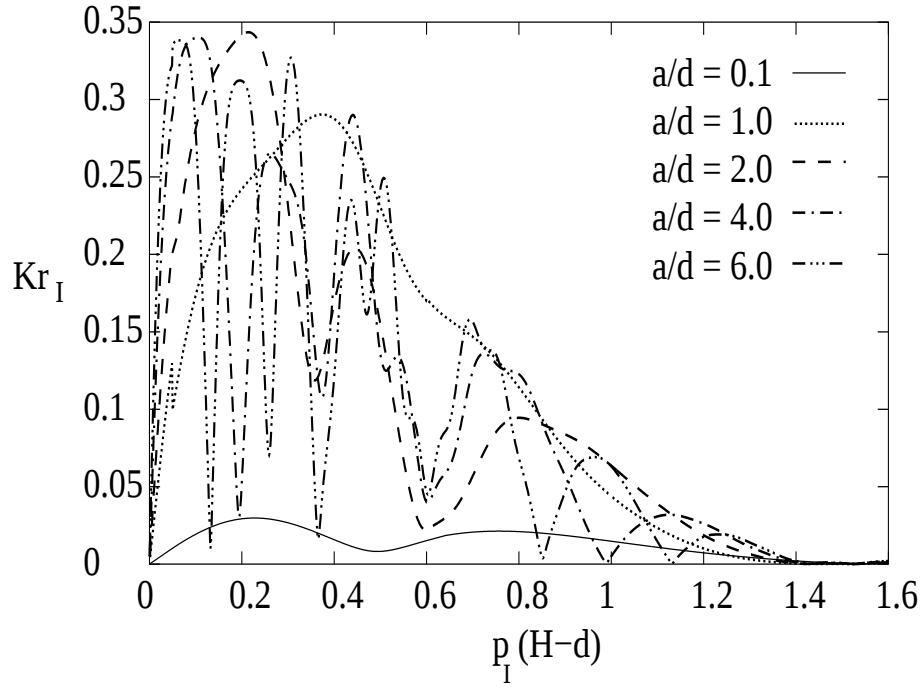


(a)

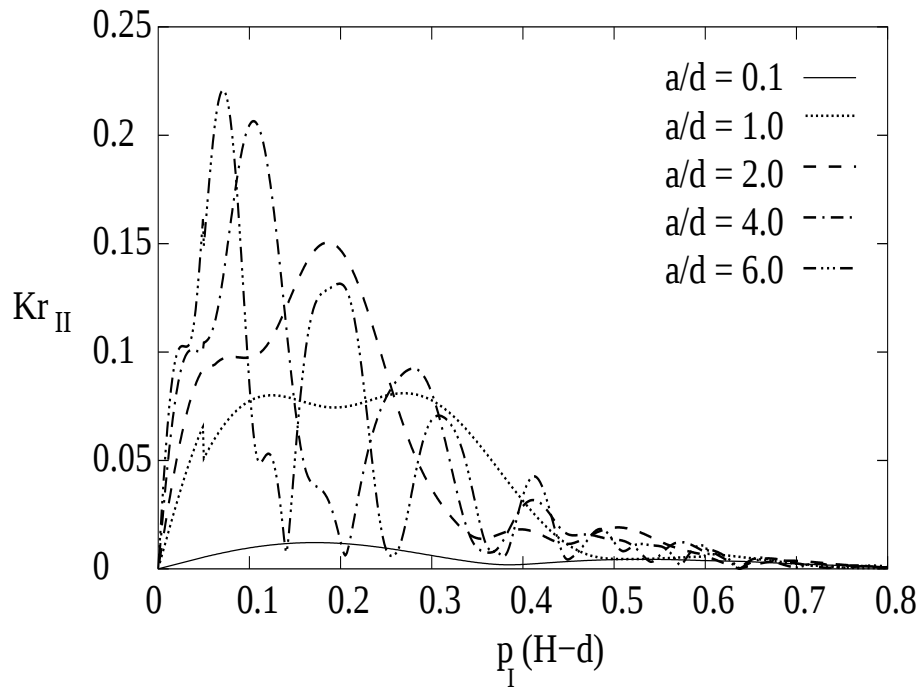


(b)

Figure 5.3: Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I(H-d)$ for a single bottom-standing dike at different H/d values, $a/d = 6.0$, $h/H = 0.25$ and $s = 0.75$.

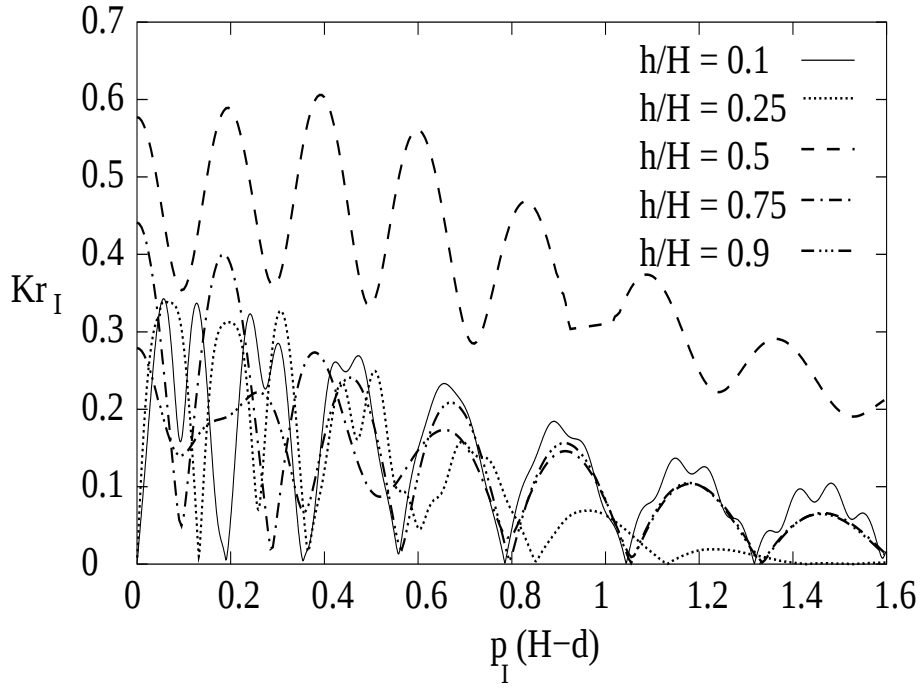


(a)

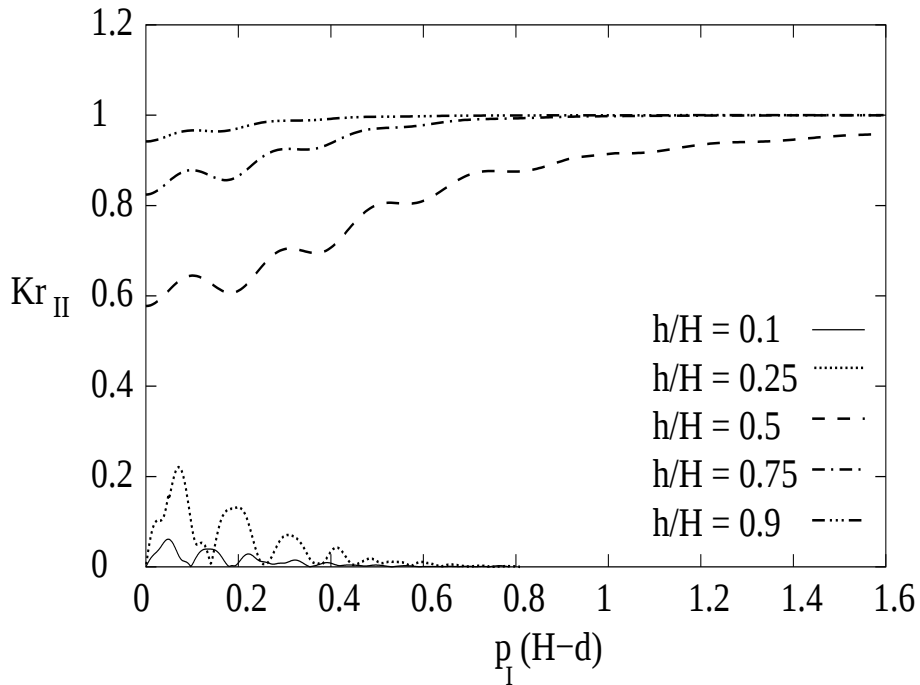


(b)

Figure 5.4: Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I(H-d)$ for a single bottom-standing dike at different a/d values, $H/d = 2.0$, $h/H = 0.25$ and $s = 0.75$.

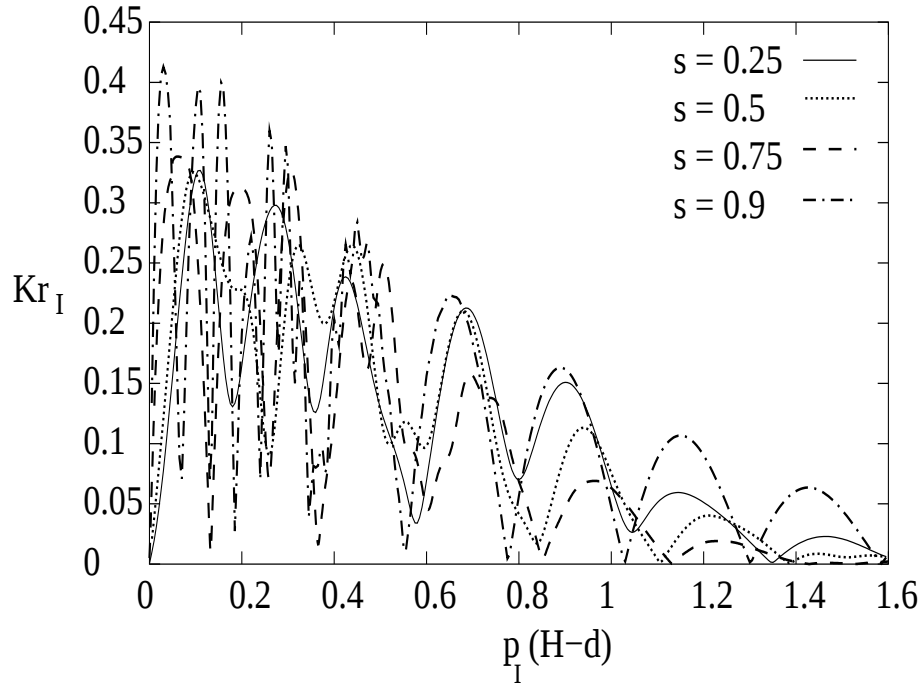


(a)

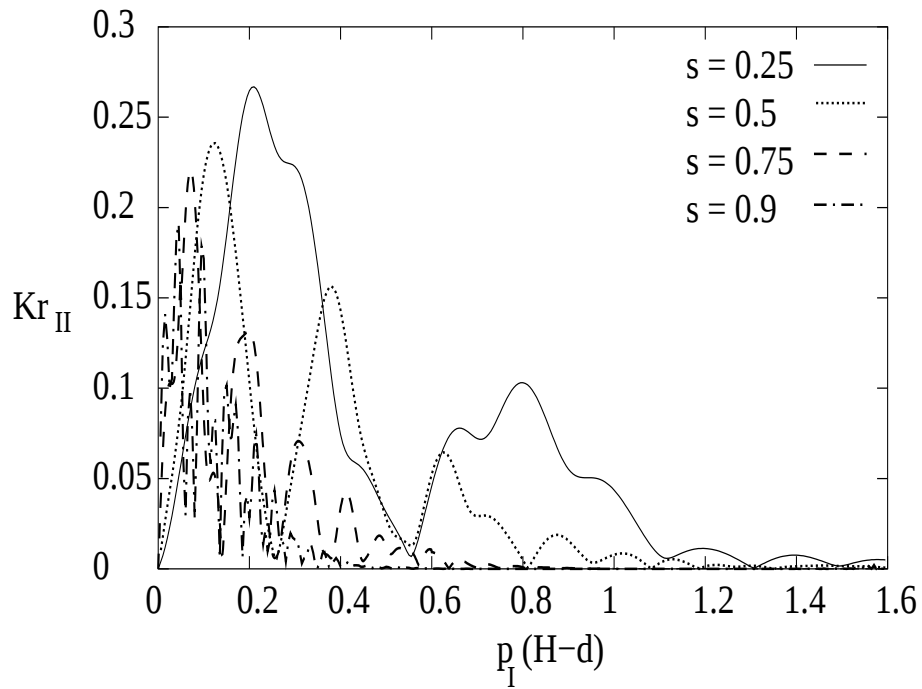


(b)

Figure 5.5: Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I(H-d)$ for a single bottom-standing dike at different h/H values, $H/d = 2.0$, $a/d = 6.0$ and $s = 0.75$.

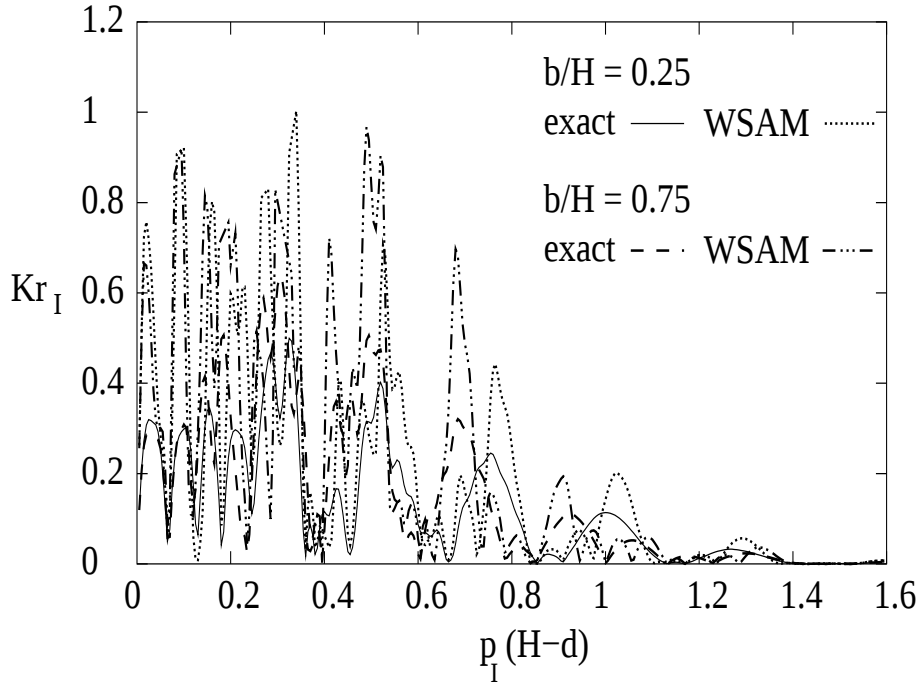


(a)

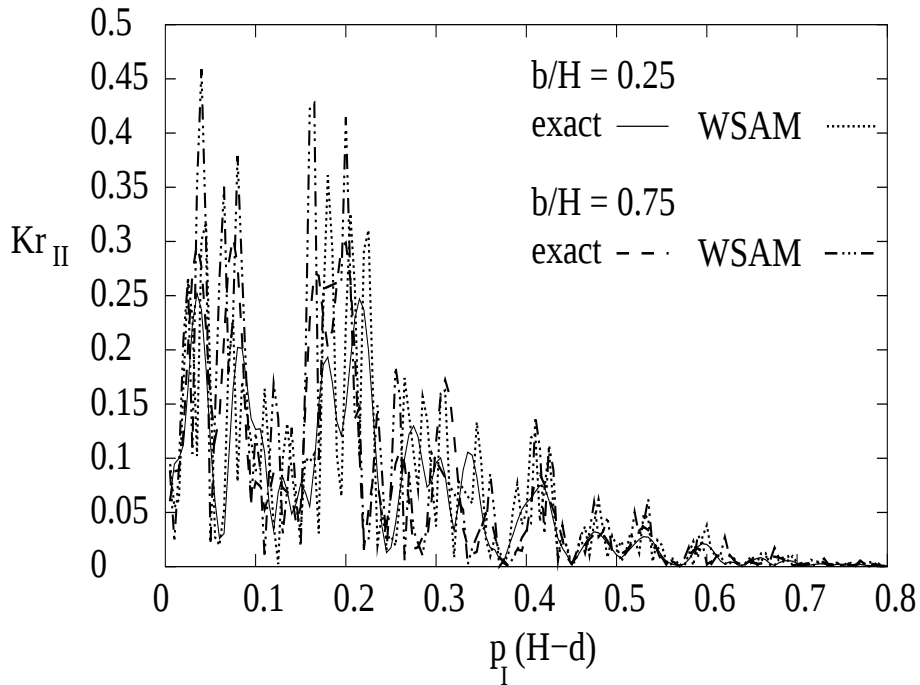


(b)

Figure 5.6: Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I(H-d)$ for a single bottom-standing dike at different s values, $H/d = 2.0$, $a/d = 6.0$ and $h/H = 0.25$.



(a)



(b)

Figure 5.7: Reflection coefficients in (a) SM, Kr_I and (b) IM, Kr_{II} versus $p_I d$ for a pair of identical surface-piercing dikes at different b/H values, $H/d = 6.0$, $a/d = 1.0$, $s = 0.75$ and $h/H = 0.25$.

Chapter 6

WAVE PAST POROUS MEMBRANE BREAKWATER

6.1 INTRODUCTION

After analyzing the studies on rigid rectangular surface-piercing and bottom-standing dikes, wave scattering by flexible porous breakwaters are considered. Wave past flexible porous structures involves relatively complex flow physics and difficult mathematical formulation. In the present chapter, scattering of water waves by a flexible porous membrane breakwater in a two-layer fluid having a free surface is analyzed. Similar to the problems considered in the past and in the sequel, the present problem is also formulated based on the usual 2D potential flow assumption with time harmonic potential.

6.2 DEFINITION OF THE PHYSICAL PROBLEM

In wave scattering by flexible porous membrane in a two-layer fluid (Fig. 6.1), the fluids are separated by a common interface (undisturbed surface located at $y = h$). The upper fluid has a free surface (undisturbed surface located at $y = 0$), and each fluid is of infinite horizontal extent ($-\infty < x < +\infty$); both the upper and lower fluid are of finite depth, $0 < y < h$ and $h < y < H$ respectively. Region 1 is defined as $-\infty < x < 0$, $0 < y < H$

and region 2 is defined as $0 < x < +\infty$; $0 < y < H$ (see Fig. 6.1). The porous membrane is located at $x = 0$, $0 < y < H$.

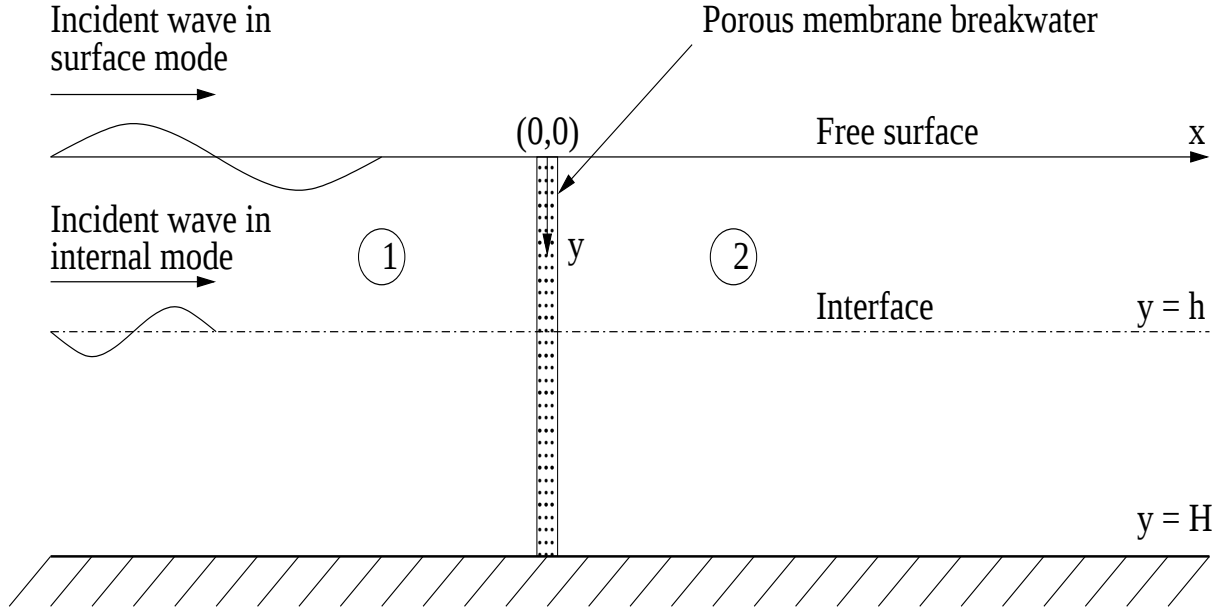


Figure 6.1: Definition sketch for flexible porous membrane breakwater.

6.3 MODEL FOR FLUID FLOW

Considering the waves incident from large negative x upon the flexible porous membrane, the velocity potentials are obtained by eigenfunction-expansion method, similar to the case of surface-piercing dike, in each of the two regions 1 and 2 as marked in Fig. 6.1. The present fluid flow problem is a boundary value problem which satisfies the fluid flow governing equation (3.1) along with the conditions Eqs. 3.12 – 3.14 and Eqs. 3.16 – 3.18. The velocity potentials in the regions 1 and 2 are given by

$$\phi_1 = \left(\sum_{n=I}^{II} I_n e^{ip_n x} + \sum_{n=I,II,1}^{\infty} R_n e^{-ip_n x} \right) f_n(p_n, y), \quad \text{for } x < 0, \quad (6.1)$$

and

$$\phi_2 = \sum_{n=I,II,1}^{\infty} T_n e^{ip_n x} f_n(p_n, y), \quad \text{for } x > 0. \quad (6.2)$$

The eigenfunctions f_n 's are given by

$$f_n(p_n, y) = \begin{cases} \frac{\sinh p_n(H-h) [p_n \cosh p_n y - K \sinh p_n y]}{K \cosh p_n h - p_n \sinh p_n h}, & \text{for } 0 < y < h, \\ \cosh p_n(H-y), & \text{for } h < y < H, \end{cases} \quad (n = I, II, 1, 2, \dots) \quad (6.3)$$

where, R_n and T_n for $n = I, II, 1, 2, 3, \dots$ are unknown constants to be determined.

Note that in the present problem only open water regions exist. Hence, the wave numbers p_n ($n = I, II$ for positive real roots and $n = 1, 2, 3, \dots$ for positive purely imaginary roots) are the roots of the dispersion relation in p as given in Eq. 4.8.

Similar to the case of single surface-piercing dikes, in the present problem the open water region eigenfunctions f_n 's for $n = I, II, 1, 2, 3, \dots$ are integrable in $0 < y < H$ having a single discontinuity at $y = h$ and are orthogonal (Manam and Sahoo (2005)) with respect to the inner product as defined by

$$\langle f_n, f_m \rangle_4 = s \int_0^h f_n f_m dy + \int_h^H f_n f_m dy. \quad (6.4)$$

The reflection and transmission coefficients in SM and IM are defined by (Manam and Sahoo (2005))

$$\begin{aligned} Kr_I &= \left| \frac{R_I}{I_I} \right| \text{ and } Kt_I = \left| \frac{T_I}{I_I} \right| \text{ in SM,} \\ Kr_{II} &= \left| \frac{R_{II}}{I_{II}} \right| \text{ and } Kt_{II} = \left| \frac{T_{II}}{I_{II}} \right| \text{ in IM.} \end{aligned} \quad (6.5)$$

6.4 MODEL FOR MEMBRANE RESPONSE

As described earlier in the general mathematical formulation, it is assumed that break-water is deflected horizontally with displacement $\zeta(y, t) = Re[\xi(y)e^{-i\omega t}]$, where $\xi(y)$ represents the complex deflection amplitude and is assumed to be small as compared to the water depth. It is assumed that the membrane is a thin, homogeneous and inextensible sheet with uniform mass m_s ($m_s = \rho_s b$, b is the thickness of the membrane, ρ_s is the uniform membrane mass density) under constant tension T . With these assumptions, the governing equation (Eq. 3.21) relating the membrane displacement ξ from equilibrium to

that of differential pressure acting on the membrane at $x = 0$ can be applied. This gives

$$\frac{d^2\xi}{dy^2} + \beta^2\xi = \begin{cases} \frac{i\omega\rho_1}{T}(\phi_2 - \phi_1), & \text{for } 0 < y < h, \\ \frac{i\omega\rho_2}{T}(\phi_2 - \phi_1), & \text{for } h < y < H, \end{cases} \quad (6.6)$$

where $\beta = \omega\sqrt{m_s/T}$ is the breakwater frequency parameter.

The membrane is pinned at the free surface and at the bottom. The corresponding boundary conditions as given in Eq. 3.24 can be applied. This gives

$$\xi(0) = 0, \quad \xi(H) = 0. \quad (6.7)$$

The continuity condition across the interface (Eq. 3.30) should be imposed. Hence the continuity of deflection and slope of the membrane breakwater across the interface (the point on the breakwater where the two fluid meet each other ($x = 0$; $y = h$)) yield

$$\xi(h^-) = \xi(h^+), \quad \xi'(h^-) = \xi'(h^+). \quad (6.8)$$

Applying the porous boundary condition (Eq. 3.36) on the porous membrane breakwater, we obtain

$$\frac{\partial\phi_j}{\partial x} = ik_0G(\phi_1 - \phi_2) + i\omega\xi \quad (j = 1, 2) \quad \text{on } x = 0, 0 < y < H, \quad (6.9)$$

where G is the complex porous-effect parameter as defined in Eq. 3.37.

6.5 GENERAL SOLUTION PROCEDURE

Applying the continuity of ϕ_x (Eq. 6.9) along the porous breakwater on $x = 0$ and invoking the orthogonality relation (Eq. 6.4) over $(0 < y < h) \cup (h < y < H)$, we obtain

$$I_n - R_n = T_n \text{ for } n = I, II \text{ and } R_n = -T_n \text{ for } n = 1, 2, 3, \dots. \quad (6.10)$$

Utilizing the Eqs. 6.1, 6.2 and 6.10 a general solution is obtained for the second order non-homogeneous ODE, Eq. 6.6 (membrane breakwater governing equation) and is given by

$$\xi(y) = C'e^{i\beta y} + C''e^{-i\beta y} - \frac{2i\omega\rho}{T} \sum_{n=I,II,1}^{\infty} \frac{R_n}{p_n^2 + \beta^2} f_n(p_n, y) \quad \text{for } 0 < y < H, \quad (6.11)$$

where the arbitrary constants C' , C'' and the fluid density ρ are defined as below.

$$C' = \begin{cases} C_1 & \text{for } 0 < y < h, \\ C_3 & \text{for } h < y < H, \end{cases} \quad C'' = \begin{cases} C_2 & \text{for } 0 < y < h, \\ C_4 & \text{for } h < y < H, \end{cases} \quad \rho = \begin{cases} \rho_1 & \text{for } 0 < y < h, \\ \rho_2 & \text{for } h < y < H. \end{cases} \quad (6.12)$$

Substituting the general solution for ξ (Eq. 6.11) in Eq. 6.9 and using the relations in Eqs. 6.1, 6.2 and 6.10 the following expression is derived.

$$h_0(y) - \sum_{n=I,II,1}^{\infty} R_n h_n(y) = 0, \quad 0 < y < H, \quad (6.13)$$

where

$$h_0(y) = \begin{cases} ip_I I_I f_I(p_I, y) + ip_{II} I_{II} f_{II}(p_{II}, y) - i\omega C_1 e^{i\beta y} - i\omega C_2 e^{-i\beta y}, & \text{for } 0 < y < h, \\ ip_I I_I f_I(p_I, y) + ip_{II} I_{II} f_{II}(p_{II}, y) - i\omega C_3 e^{i\beta y} - i\omega C_4 e^{-i\beta y}, & \text{for } h < y < H, \end{cases} \quad (6.14)$$

and

$$h_n(y) = \begin{cases} \left[\frac{2\omega^2 \rho_1}{(p_n^2 + \beta^2)T} + ip_n + 2ip_I G \right] f_n(p_n, y), & \text{for } 0 < y < h, \\ \left[\frac{2\omega^2 \rho_2}{(p_n^2 + \beta^2)T} + ip_n + 2ip_I G \right] f_n(p_n, y), & \text{for } h < y < H, \end{cases} \quad (n = I, II, 1, 2, \dots) \quad (6.15)$$

We can apply the least-squares-approximation method to Eq. 6.13 as described earlier in general mathematical formulation chapter. We can write

$$Q(y) = h_0(y) - \sum_{n=I,II,1}^N R_n h_n(y), \quad \text{for } 0 < y < H. \quad (6.16)$$

Applying the least-squares method, we obtain

$$\int_0^H \bar{\mathbf{Q}}(y) \frac{\partial Q(y)}{\partial R_n} dy = 0, \quad \text{for } n = I, II, 1, 2, \dots, N, \quad (6.17)$$

where the bar denotes the complex conjugate.

Eq. 6.17 provides $N + 2$ linear equations with $N + 6$ unknowns, as $h_0(y)$ involves 4 extra unknowns C_1 , C_2 , C_3 and C_4 . Substituting the expression for ξ (Eq. 6.11) in the end conditions on the breakwater as in Eq. 6.7 and the continuity conditions at the

interface as in Eq. 6.8 yield the required another 4 linear equations. These system of equations are solved using Gauss-elimination method to compute and analyze various physical quantities of interest. Number of evanescent modes in the series are selected based on the experience of the numerical convergence experiment.

6.6 NUMERICAL RESULTS AND DISCUSSION

In the present section, numerical results on the combined effect of porosity and membrane tension are discussed, to analyze the performance of membrane breakwater in the two-layer fluid for various non-dimensional parameters. The wave and membrane parameters are given in terms of non-dimensional values of wave number $p_I H$, water depth h/H , fluid density ratio s , porous-effect parameter G , membrane tension $T' = T/(\rho_1 g h^2)$, and membrane mass $m' = m_s/\rho_1 h$. The membrane mass m' is kept fixed ($m' = 0.1$) throughout the analysis as the effect of membrane mass on the performance characteristic of the breakwater is insignificant (Kim and Kee (1996), and Lo (1998)).

6.6.1 Reflected and Transmitted Energy

The wave transmission across a permeable flexible breakwater is governed by two combined phenomena. When a train of waves approaches a permeable flexible breakwater, seepage flow induced by waves penetrates through the breakwater and waves are reproduced with some dissipation after transmission. On the other hand, due to deformation of the flexible breakwater, the waves are regenerated in the downstream side, even if there is no flow across the breakwater.

In general, the energy reflection and transmission provide one of the major criteria in deciding the effectiveness of the breakwater. In this subsection, the effect of various non-dimensional physical parameters on energy reflection and transmission in both SM and IM are analyzed. For the sake of simplicity, all results in the present subsection are analyzed with respect to the normalized SM wave number $p_I H$ by allowing the normalized IM wave number $p_{II} H$ to vary based on the two-layer fluid dispersion relation. It is observed from

the general trend of wave reflection in SM that the wave reflection decreases from its peak to a certain value in the shallow water region and thereafter it attains a constant value. On the other hand, the wave reflection in IM increases from zero to a certain value in the shallow water region and thereafter it attains a constant value (see Figs. 6.2 – 6.5). Similar results are obtained for wave reflection by a flexible membrane breakwater in a single-layer fluid by Lo (2000) and Lee and Lo (2002) (see Fig. 3 (a) of Lo (2000) and Fig. 5 of Lee and Lo (2002)). Furthermore, the wave reflection in SM is found to be significantly smaller than the wave reflection in IM, which suggests that a membrane breakwater is more effective in IM wave motion than in SM wave motion. Similar observations for porous breakwaters in a two-layer fluid are reported by Manam and Sahoo (2005).

In Fig. 6.2 (a) and (b), the reflection and transmission coefficients in SM and IM respectively are plotted against $p_I H$, for different values of membrane tension parameter T' . It is observed that higher wave transmission and lower wave reflection occur in SM where as lower wave transmission and higher wave reflection occur in IM over the range of practical interest.

The variation of reflection and transmission coefficients versus $p_I H$ for both SM and IM are plotted in Fig. 6.3 (a) and (b) respectively for different values of the porous-effect parameter G . In general, the wave reflection in both SM and IM increases with a decrease in the value of $|G|$ and a reverse trend is observed in the case of wave transmission. This is expected, because an increase in porosity not only allows more waves to pass through the breakwater but also reduces the membrane breakwater resistance to the wave motion.

The effect of non-dimensional water depth h/H of two fluids on the reflection and transmission coefficients in SM and IM are shown in Fig. 6.4 (a) and (b) respectively. In SM wave motion it is observed that the wave transmission is lower and the wave reflection is higher for a thinner upper layer i.e., for $h/H = 0.25$ (Fig. 6.4 (a)). However, except for very small values of $p_I H$ the wave reflection and transmission are same for $h/H = 0.5$ and 0.75 . On the other hand, an opposite trend is observed in case of IM wave motion where the wave transmission is higher and the wave reflection is lower for a thinner upper layer i.e., $h/H = 0.25$ (Fig. 6.4 (b)) almost over the entire range of interest. This may be

due to the resonating interaction induced by vertical flexible breakwater, between surface- and internal-waves, when the free surface is close to the interface.

The reflection and transmission coefficients versus $p_I H$ are plotted in SM and IM for different fluid density ratio s in Fig. 6.5 (a) and (b) respectively. In Fig. 6.5 (a) it is observed that the fluid density ratio s has negligible effect on both wave reflection and transmission for SM wave motion. However, the wave reflection in SM is observed to be marginally higher for large value of s ($s = 0.75$). On the other hand, the wave reflection increases and the wave transmission decreases for IM wave motion with an increase in fluid density ratio (Fig. 6.5 (b)). This nature of the wave transmission in IM may be due to the high interface elevation as the fluid density ratio s approaches to unity (Kundu and Cohen (2002) and Milne-Thomson (1996)).

6.6.2 Free Surface and Interface Elevations

The nature of free surface elevation η_{fs} and interface elevation η_{int} versus non-dimensional distance x/λ_I are studied after normalizing with respect to the amplitude of the incident waves in the surface mode. This normalization gives a clear understanding about the amplitude of the free surface elevation to that of interfacial wave elevation. The free surface and interface elevations near the breakwater are the result of mutual interaction of propagating and evanescent modes of both surface and internal-waves (see Figs. 6.6 – 6.8). Hence the free surface and interface elevations in a two-layer fluid are combinations of two prominent wave patterns which are referred to as primary and secondary wave patterns in the present paper. The primary pattern is the one which is generated due to SM wave motion and the secondary wave pattern is that developed due to the IM wave motion. In general, it is observed that the interface elevation is much larger than that of the free surface elevation when either the densities of the two fluids are very close or in the case when the interface and free surface are close to each other. A similar situation exists in a real ocean, as explained theoretically in Milne-Thomson (1996) (see page 445). One of the reasons for such a high wave amplitude may be due to the resonating interaction between the waves in SM and IM.

Fig. 6.6 (a) and (b) show the pattern of the free surface and interface elevation respectively for different values of membrane tension parameter T' . The effect of change in tension T' is significant only near the locations of local maxima and minima of the secondary wave pattern in the case of free surface elevation Fig. 6.6 (a). On the other hand, the interface elevation is found to be independent of the variation in membrane tension (see Fig. 6.6 (b)).

Variation of free surface and interface elevation at different h/H ratios are shown in Fig. 6.7 (a) and (b) respectively. It is observed that as the interface and free surface become nearer, the amplitudes of both free surface and interface elevations becomes higher. This may be due to the resonating interaction between the waves in SM and IM. The magnitude of the primary and secondary wave pattern amplitudes of the free surface elevation are of same order for small h/H ratio (Fig. 6.7 (a)). This is due to the fact that the interface elevation increases rapidly when free surface and interface are close to each other (see Fig. 6.7 (b)).

Fig. 6.8 (a) and (b) show the pattern of the free surface and interface elevations for different fluid density ratios s . It is observed that the amplitude of the free surface elevation increases with decrease in the fluid density ratio s (Fig. 6.8 (a)). On the other hand, an opposite trend is observed in case of interface elevation where amplitudes of the interface increases with an increase in fluid density ratio. As the fluid density ratio s approaches one, the secondary wave pattern of the free surface and the interface elevations amplify rapidly, which is a well known phenomenon in the case of inter-facial waves (see Kundu and Cohen (2002) and Milne-Thomson (1996)). It is important to note that among the elevations, the interface depends heavily on the density ratio s . The reason for this is that the amplitudes of waves in IM are very sensitive to the change in density ratio s whereas the waves in SM are least affected by the change in the value of s . Hence interface elevations change sharply with the change in parameter s whereas free surface elevations are comparatively less affected by the change in the value of s . The variation in free surface elevations with the change in s is mainly due to the existence of the secondary wave pattern, which is again caused by the internal-waves. This is the reason why, in

Fig. 6.5, the reflection and transmission coefficients in IM are more dependent on s than those in SM. Interestingly, when $s = 0.25$, the free surface elevation is free from secondary waves as in this case the internal-waves have very small amplitude. Furthermore, it is observed that with increase in the value of s the wave length of interfacial waves reduces and very short waves are observed as s approaches one.

The local effects are not visible in the elevation plots because magnitude of the contribution of local effects is insignificant as compared to that of propagating modes in SM and IM in the present study. Moreover, their contribution decays quickly as one moves away from the breakwater (either left or right) because of the exponential decay of the multiplication factor in the velocity potential. There is always a discontinuity in elevation as the waves pass the breakwater. However, in the present case the magnitude of the discontinuity is very small because the porous membrane offers very little resistance to waves. The discontinuity is only apparent in Fig. 6.8 (a) for $s = 0.25$.

6.6.3 Response of Membrane Breakwater

In the present subsection, the variation of membrane breakwater response ξ normalized with respect to incident wave amplitude I_I in SM is analyzed for various membrane and two-layer fluid parameters. In all of Figs. 6.9 – 6.12, the vanishing nature of membrane response at the two ends is because the membrane is fixed at those points.

Variation of normalized membrane response $|\xi/I_I|$ for different h/H ratios is plotted versus normalized vertical position y/H in Fig. 6.9. It is observed that the breakwater has higher deflection amplitude at a location nearer to the interface. This is due to the propagation of surface and interfacial waves at the interface in a two-layer fluid. However, the deflection is found to be higher for small h/H ratio (the interface is closer to the free surface). This is because of the higher free surface and interface elevation as observed in Fig. 6.7.

Variation of the normalized membrane response $|\xi/I_I|$ is plotted versus normalized vertical position y/H for different values of fluid density ratio s in Fig. 6.10. The membrane deflection is found to increase with the increase in fluid density ratio s . The reasons

for these observations are clear from the nature of free surface and interface elevations in Fig. 6.8 (a) and (b). However nearer to the free surface the membrane deflection in the upper fluid domain is found to be high for low fluid density ratio $s = 0.25$ as in this case the amplitude of free surface elevation is found to be quite high (see Fig. 6.8 (a)).

The normalized membrane response $|\xi/I_I|$ for various values of membrane tension parameter T' is plotted versus normalized vertical position y/H in Fig. 6.11. It is clear from Fig. 6.11 that the membrane deflection increases with decrease in membrane tension. This is expected, because a reduction in membrane tension leads to a reduction in the stiffness of the membrane against the wave motion and leads to a higher membrane deflection.

In Fig. 6.12 the normalized membrane response $|\xi/I_I|$ is plotted versus normalized vertical position y/H for different values of the porous-effect parameter G . A high membrane deflection is observed for higher values of the imaginary part of the porous-effect parameter G (the inertia effect of the fluid inside the porous breakwater).

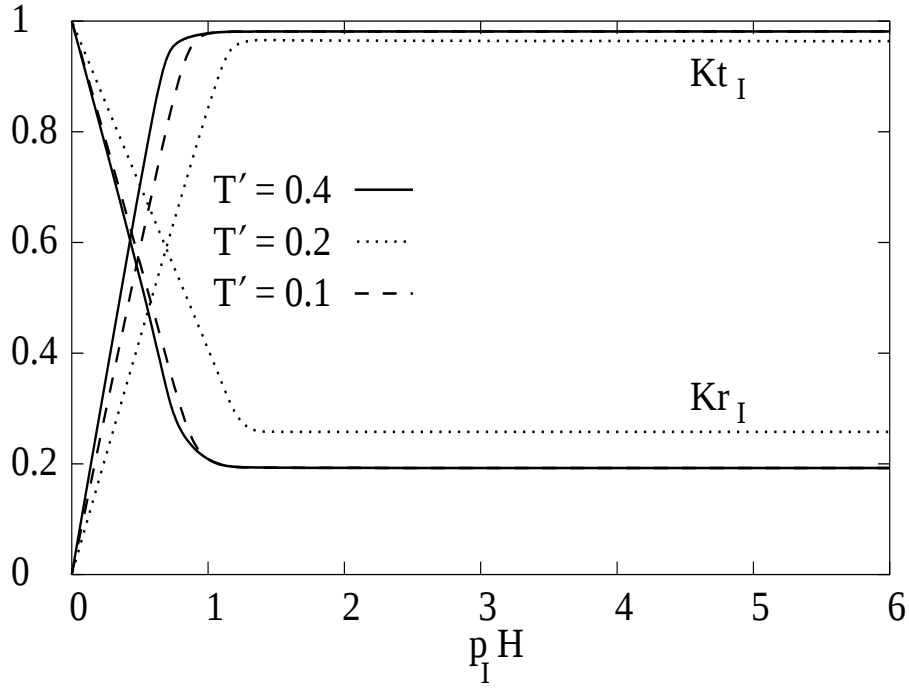
6.6.4 Summary of Important Observations

The important observations from the present numerical results for porous membrane are summarized pointwise as below:

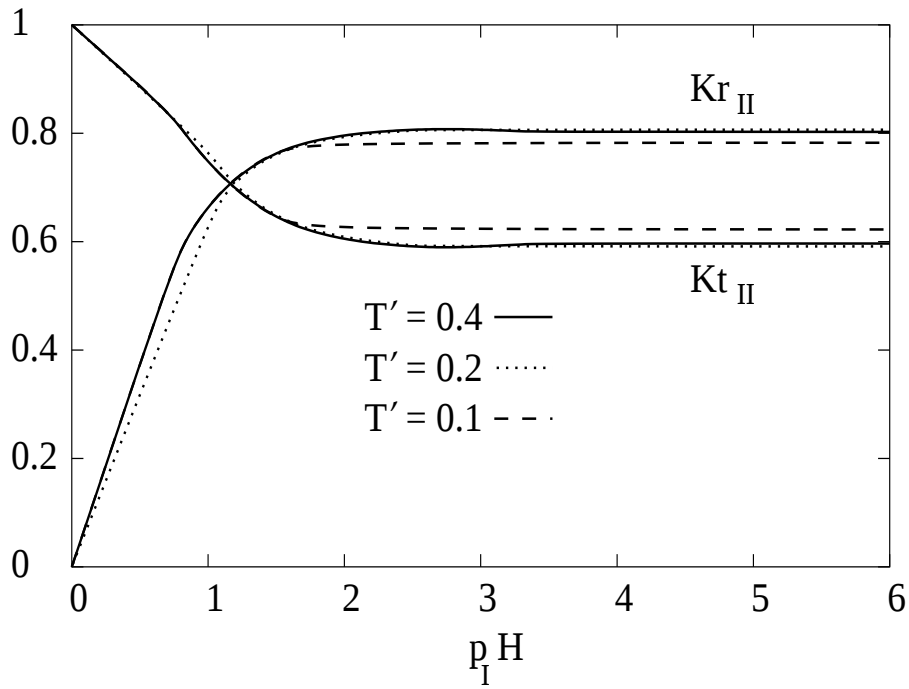
1. General trend of wave reflection in SM decreases from its peak to a certain value in the shallow water region and thereafter it attains a constant value. The wave reflection in IM increases from zero to a certain value in the shallow water region and thereafter it attains a constant value. Similar results are reported for wave reflection by a flexible membrane breakwater in a single-layer fluid (see, Fig. 3 (a) of Lo (2000) and Fig. 5 of Lee and Lo (2002)).
2. Wave reflection in SM is found to be significantly smaller than the wave reflection in IM, which suggests that a membrane breakwater is more effective in IM wave motion than in SM wave motion. Similar observations for porous breakwaters in a two-layer fluid are reported by Manam and Sahoo (2005).

3. In general, wave reflection in both SM and IM increases with a decrease in the value of porous effect parameter $|G|$ and a reverse trend is observed in the case of wave transmission.
4. In SM wave transmission is lower and the wave reflection is higher for a thinner upper layer and an opposite trend is observed in case of IM wave motion.
5. Fluid density ratio s has negligible effect on both wave reflection and transmission for SM wave motion. The wave reflection increases and the wave transmission decreases for IM wave motion with an increase in fluid density ratio.
6. Free surface and interface elevations are combinations of primary and secondary wave patterns.
7. Interface elevation is much larger than that of the free surface elevation when either the densities of the two fluids are very close or in the case when the interface and free surface are close to each other. A similar situation exists in a real ocean, as explained theoretically in Milne-Thomson (1996).
8. The effect of change in tension T' is significant only near the locations of local maxima and minima of the secondary wave pattern in the case of free surface elevation. The interface elevation is independent of the variation in membrane tension.
9. As the interface and free surface become nearer, the amplitudes of both free surface and interface elevations becomes higher.
10. Amplitude of the free surface elevation increases with decrease in the fluid density ratio s and an opposite trend is observed in case of interface elevation.
11. As the fluid density ratio s approaches one, the secondary wave pattern of the free surface and the interface elevations amplify rapidly, which is a well known phenomenon in the case of inter-facial waves (see Kundu and Cohen (2002) and Milne-Thomson (1996)).
12. Among the elevations, the interface depends heavily on the density ratio s .
13. The variation in free surface elevations with the change in s is mainly due to the existence of the secondary wave pattern, which is caused by the internal-waves.

14. With increase in the value of s the wave length of interfacial waves reduces and very short waves are observed as s approaches one.
15. Breakwater has higher deflection amplitude at a location nearer to the interface.
16. Breakwater deflection is higher for small h/H ratio (the interface is closer to the free surface).
17. Membrane deflection increases with the increase in fluid density ratio s .
18. Nearer to the free surface the membrane deflection in the upper fluid domain is found to be high for low fluid density ratio $s = 0.25$ as in this case the amplitude of free surface elevation is found to be quite high.
19. Membrane deflection increases with decrease in membrane tension.
20. Membrane deflection is higher for high value of imaginary part of the porous-effect parameter G (the inertia effect of the fluid inside the porous breakwater).

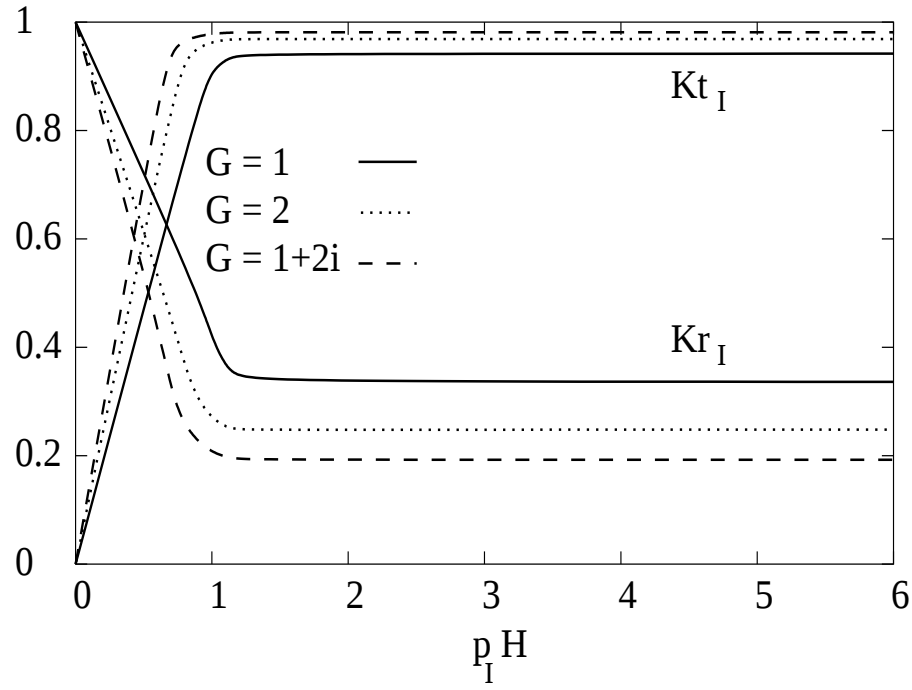


(a)

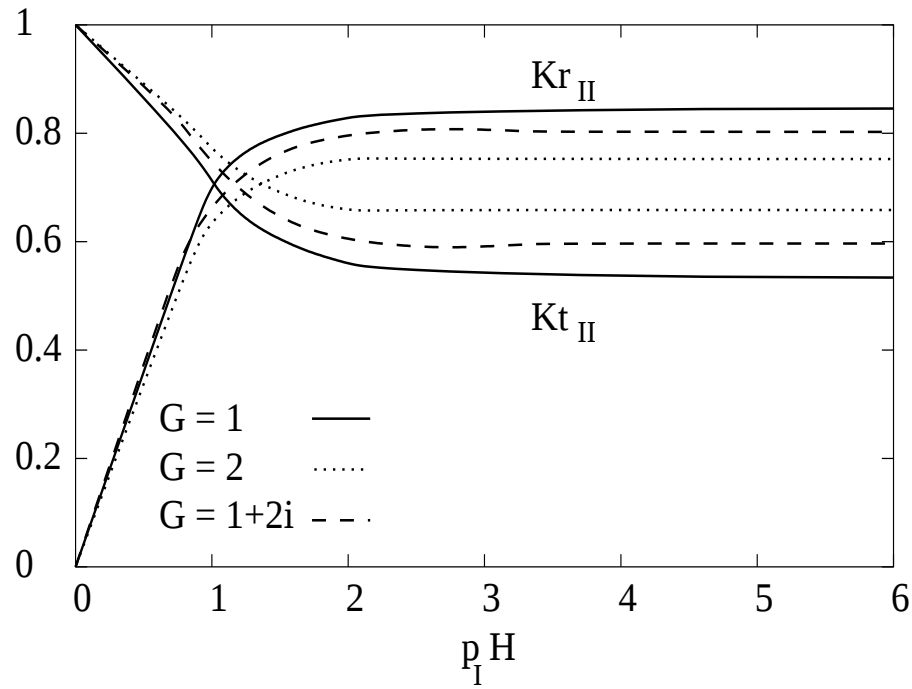


(b)

Figure 6.2: Reflection and transmission coefficients in (a) SM and (b) IM versus $p_I H$ for different T' values at $G = 1 + 2i$, $s = 0.75$ and $h/H = 0.5$.

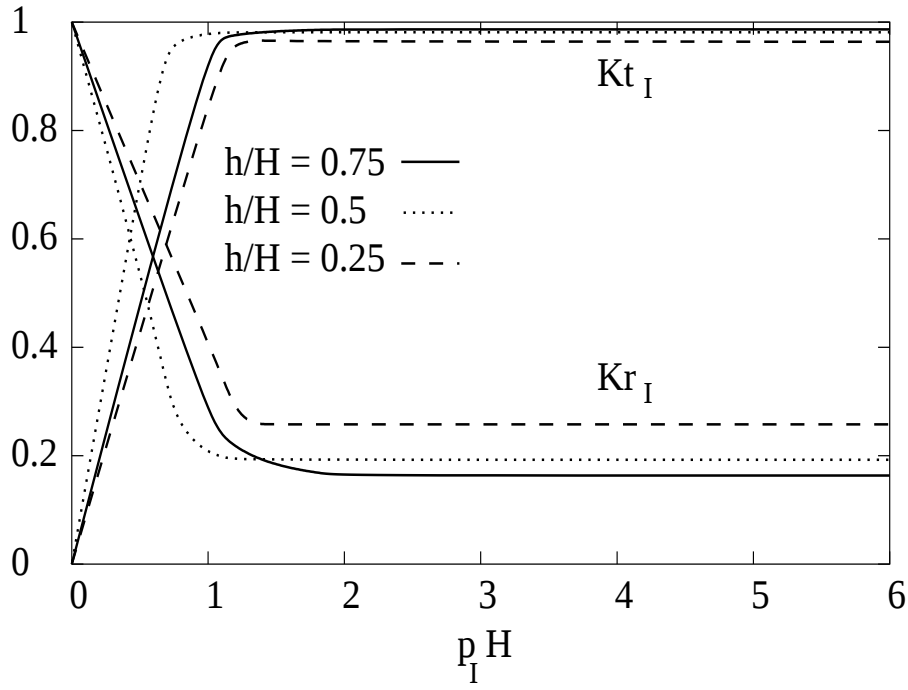


(a)

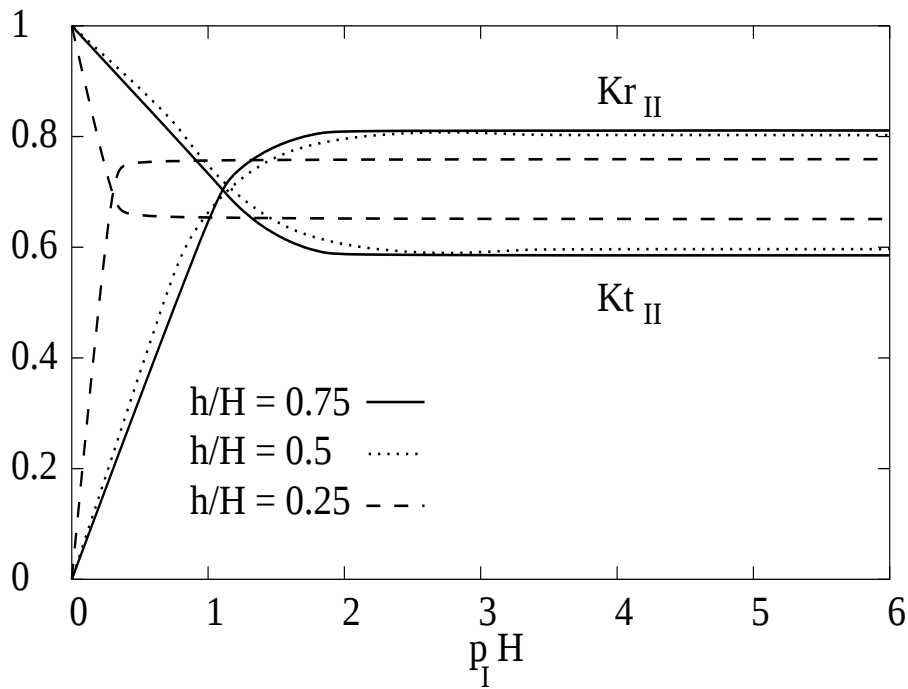


(b)

Figure 6.3: Reflection and transmission coefficients in (a) SM and (b) IM versus $p_I H$ for different G values at $h/H = 0.5$, $s = 0.75$ and $T' = 0.4$.

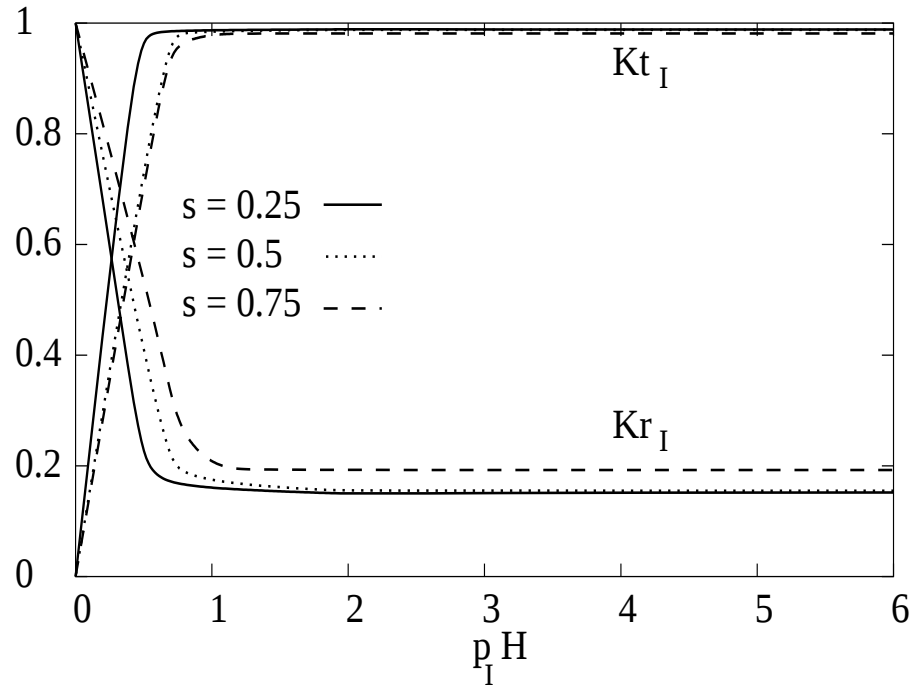


(a)

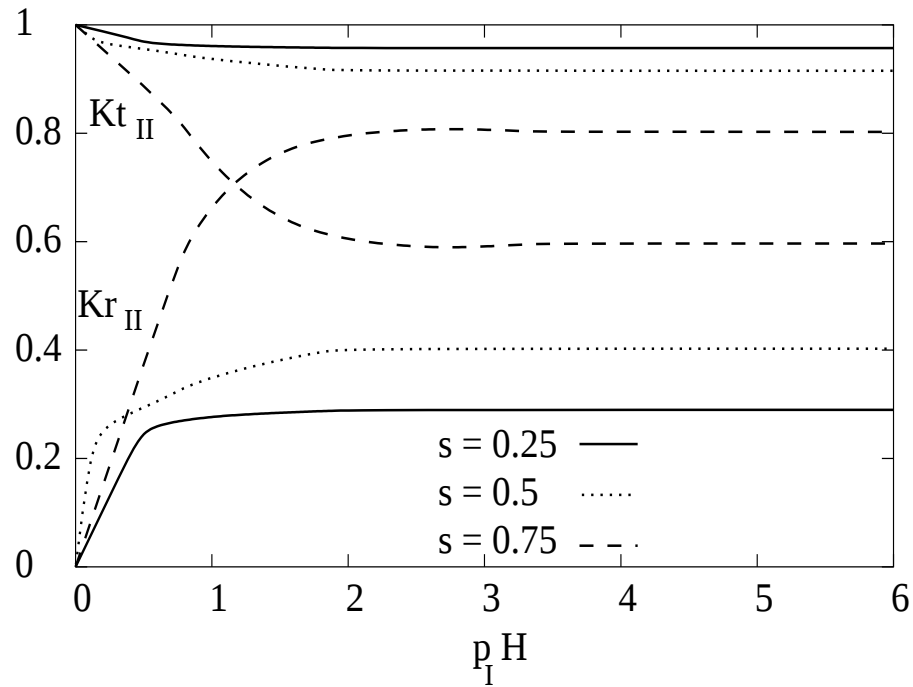


(b)

Figure 6.4: Reflection and transmission coefficients in (a) SM and (b) IM versus $p_I H$ for different h/H ratios at $G = 1 + 2i$, $s = 0.75$ and $T' = 0.4$.

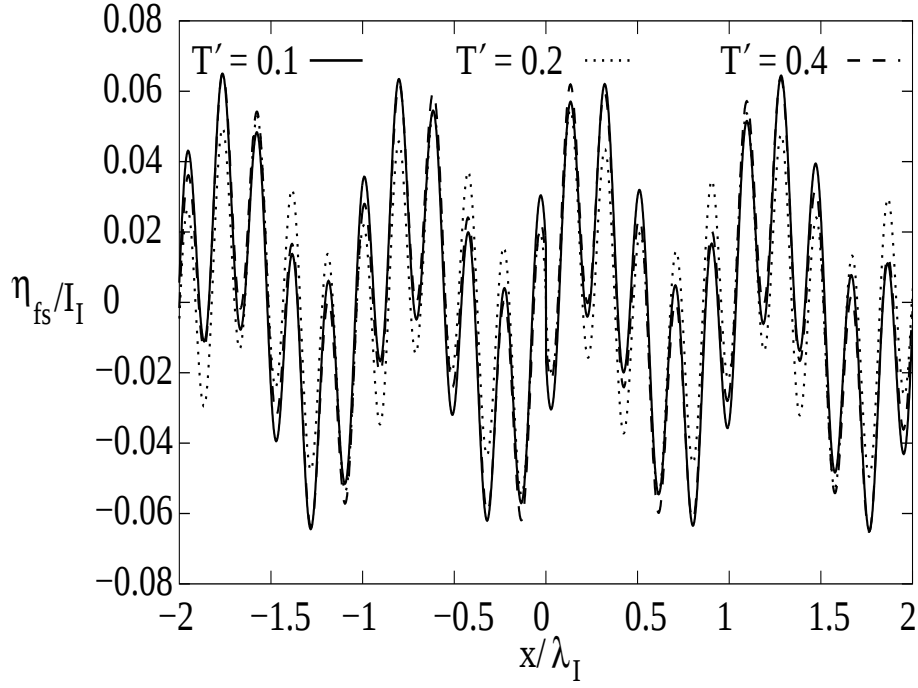


(a)

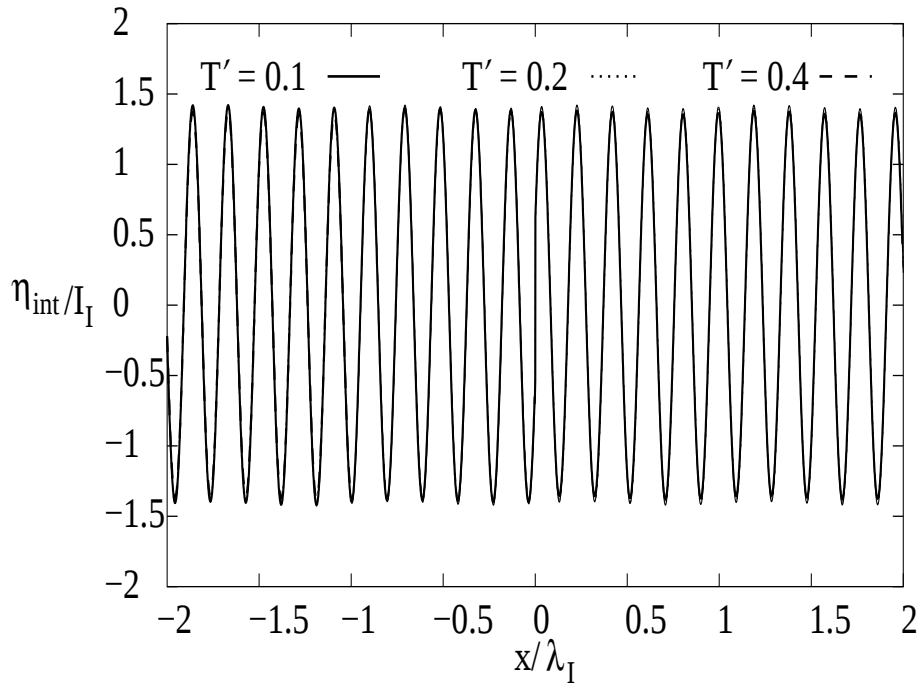


(b)

Figure 6.5: Reflection and transmission coefficients in (a) SM and (b) IM versus $p_I H$ for different s values at $h/H = 0.5$, $G = 1 + 2i$ and $T' = 0.4$.

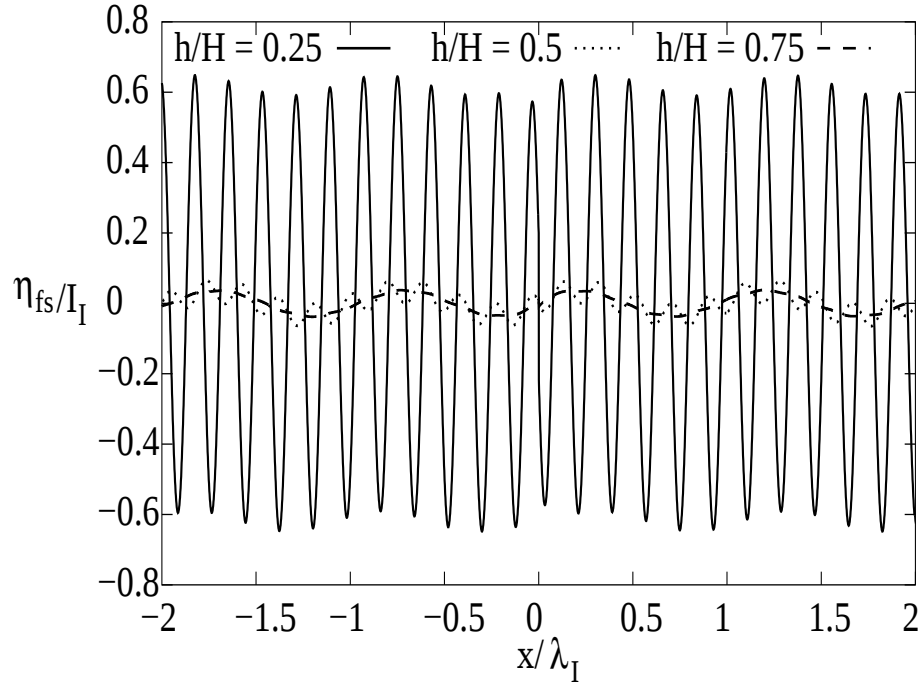


(a)

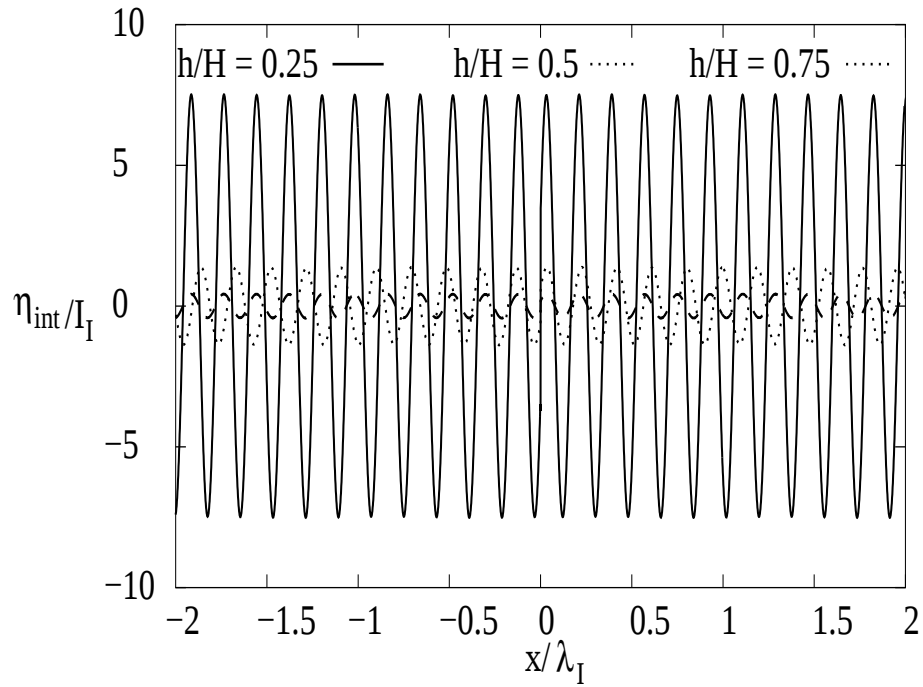


(b)

Figure 6.6: (a) Free surface and (b) Interface elevation versus x/λ_I for different T' values at $p_I H = 1.0$, $h/H = 0.5$, $G = 1 + 2i$ and $s = 0.75$.

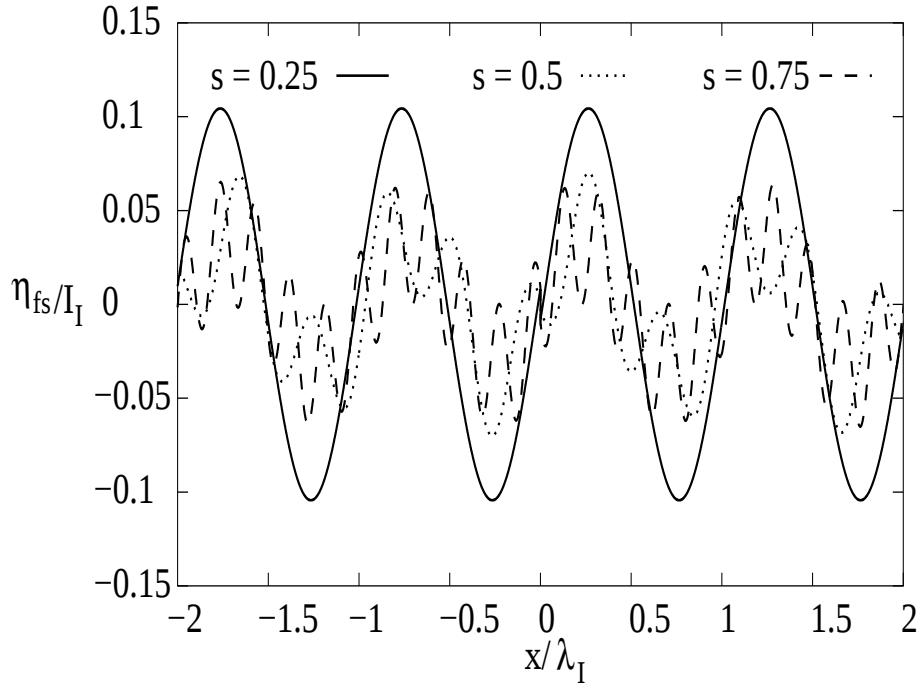


(a)

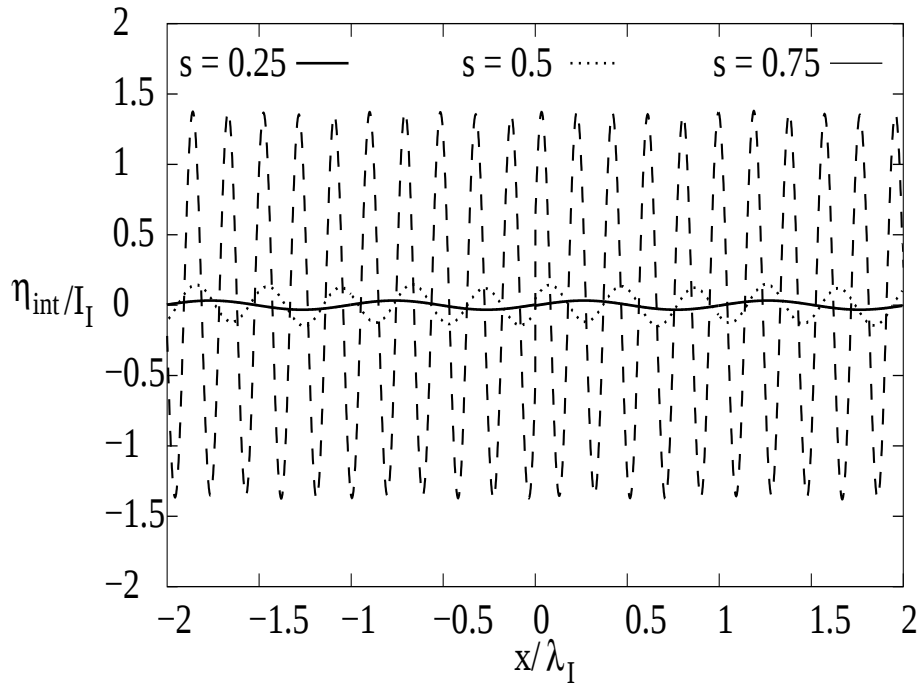


(b)

Figure 6.7: (a) Free surface and (b) Interface elevation versus x/λ_I for different h/H ratios at $p_I H = 1.0$, $G = 1 + 2i$, $s = 0.75$ and $T' = 0.4$.



(a)



(b)

Figure 6.8: (a) Free surface and (b) Interface elevation versus x/λ_I for different s values at $p_I H = 1.0$, $h/H = 0.5$, $G = 1 + 2i$ and $T' = 0.4$.

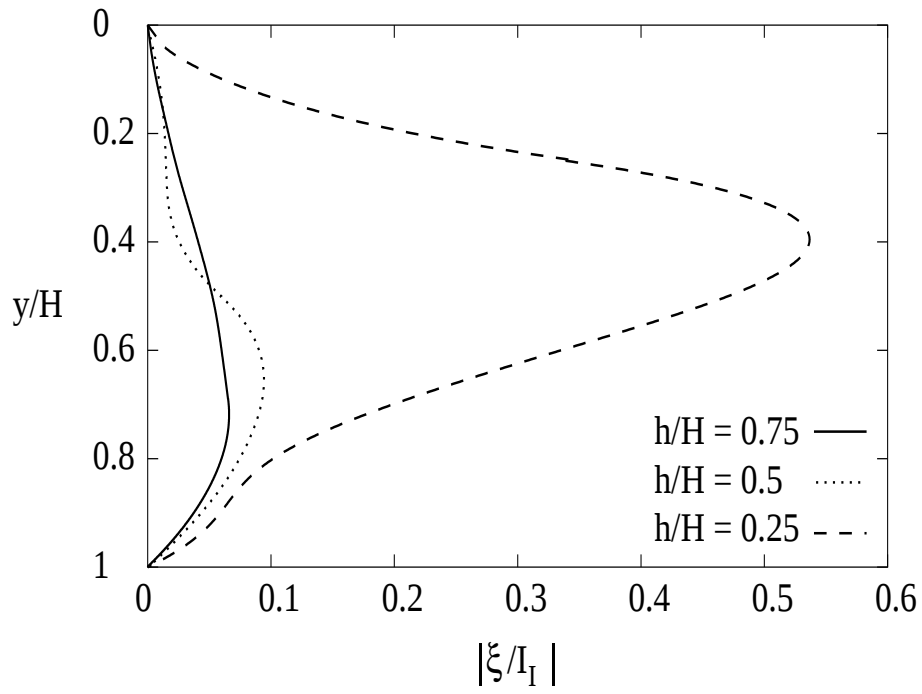


Figure 6.9: Membrane displacement versus y/H for different h/H ratios at $s = 0.75$, $p_I H = 1.0$, $T' = 0.4$ and $G = 1 + 2i$.

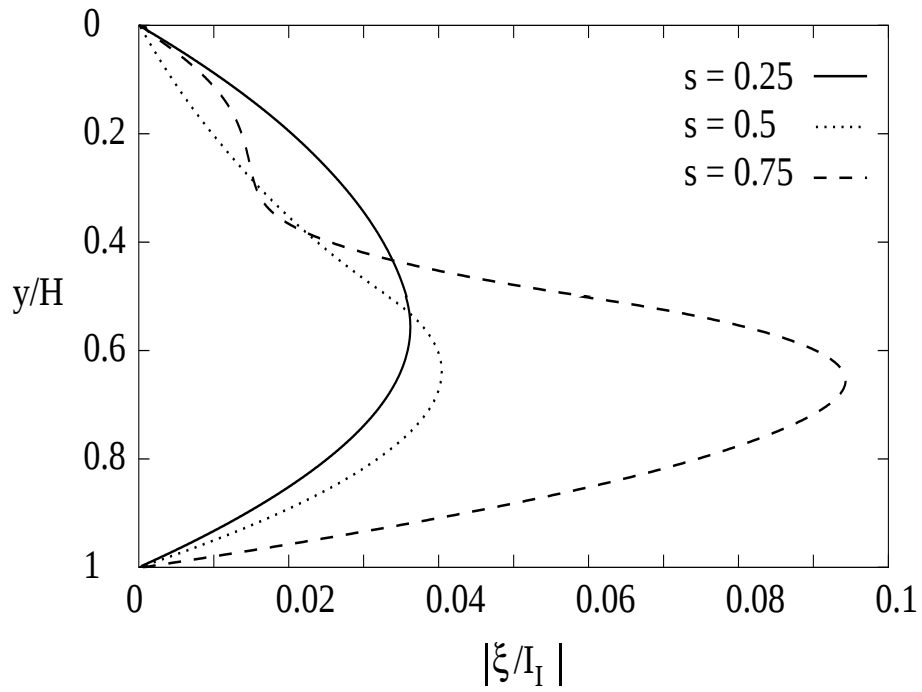


Figure 6.10: Membrane displacement versus y/H for different s values at $h/H = 0.5$, $p_I H = 1.0$, $T' = 0.4$ and $G = 1 + 2i$.

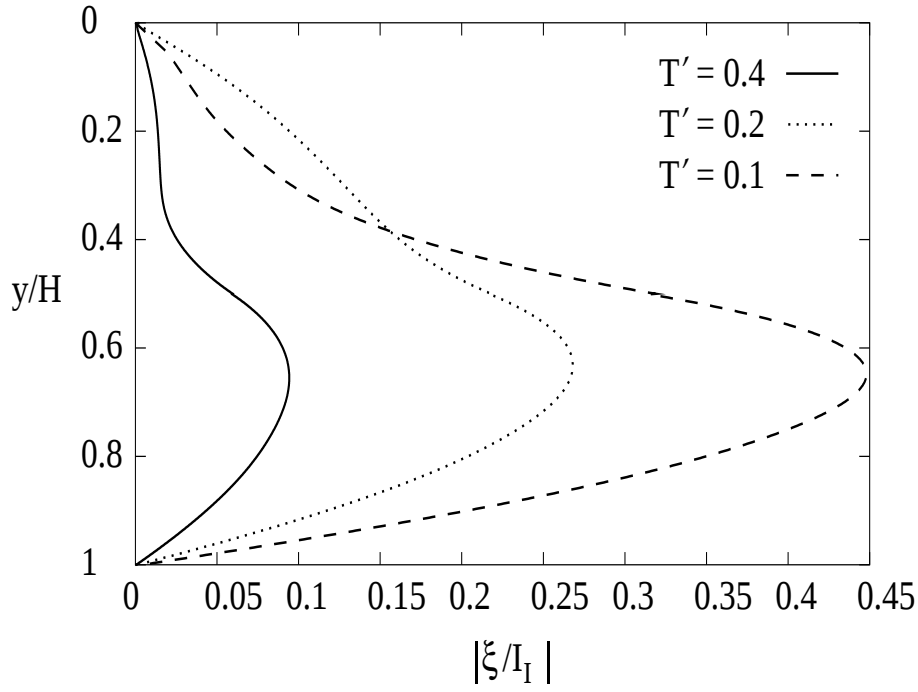


Figure 6.11: Membrane displacement versus y/H for different T' values at $h/H = 0.5$, $s = 0.75$, $p_I H = 1.0$ and $G = 1 + 2i$.

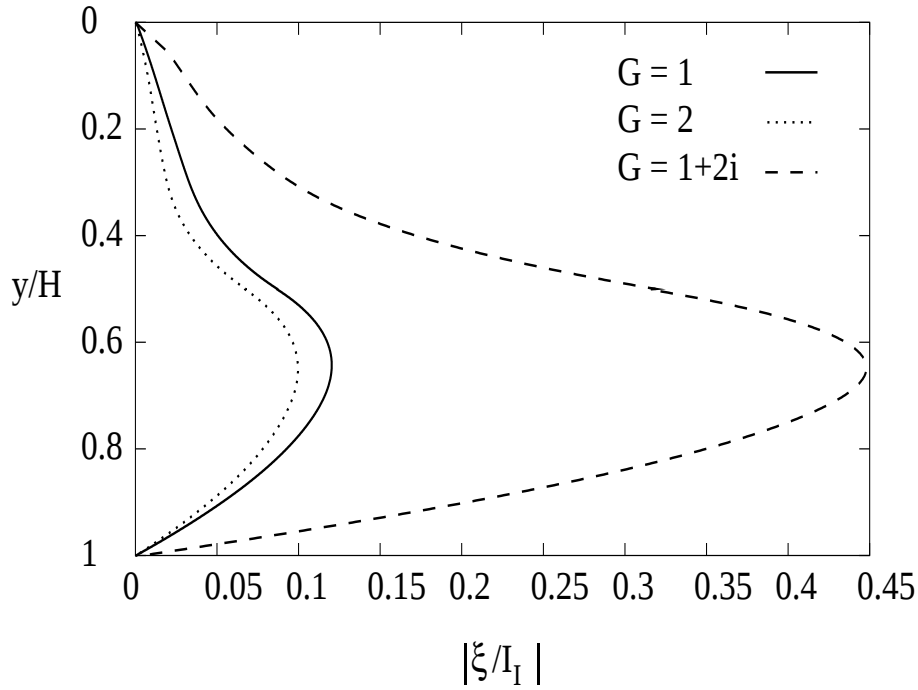


Figure 6.12: Membrane displacement versus y/H for different G values at $s = 0.75$, $p_I H = 1.0$, $h/H = 0.5$ and $T' = 0.4$.

Chapter 7

WAVE PAST POROUS PLATE BREAKWATER

7.1 INTRODUCTION

After analyzing wave past flexible porous membrane breakwater in Chapter 6, wave scattering by flexible porous plate breakwater is considered in the present chapter.

7.2 DEFINITION OF THE PHYSICAL PROBLEM

Fig. 7.1 illustrates physical problem for wave scattering by flexible porous plate in a two-layer fluid under consideration. Similar to Chapter 6, in this chapter as well, fluids are separated by a common interface (undisturbed surface located at $y = h$), wherein the upper fluid has a free surface (undisturbed surface located at $y = 0$), and each fluid is of infinite horizontal extent ($-\infty < x < +\infty$); both the upper and lower fluid are of finite depth, $0 < y < h$ and $h < y < H$ respectively. Region 1 is defined as $-\infty < x < 0$, $0 < y < H$ and region 2 is defined as $0 < x < +\infty$; $0 < y < H$ (see Fig. 7.1). The porous flexible plate is located at $x = 0$, $0 < y < H$.

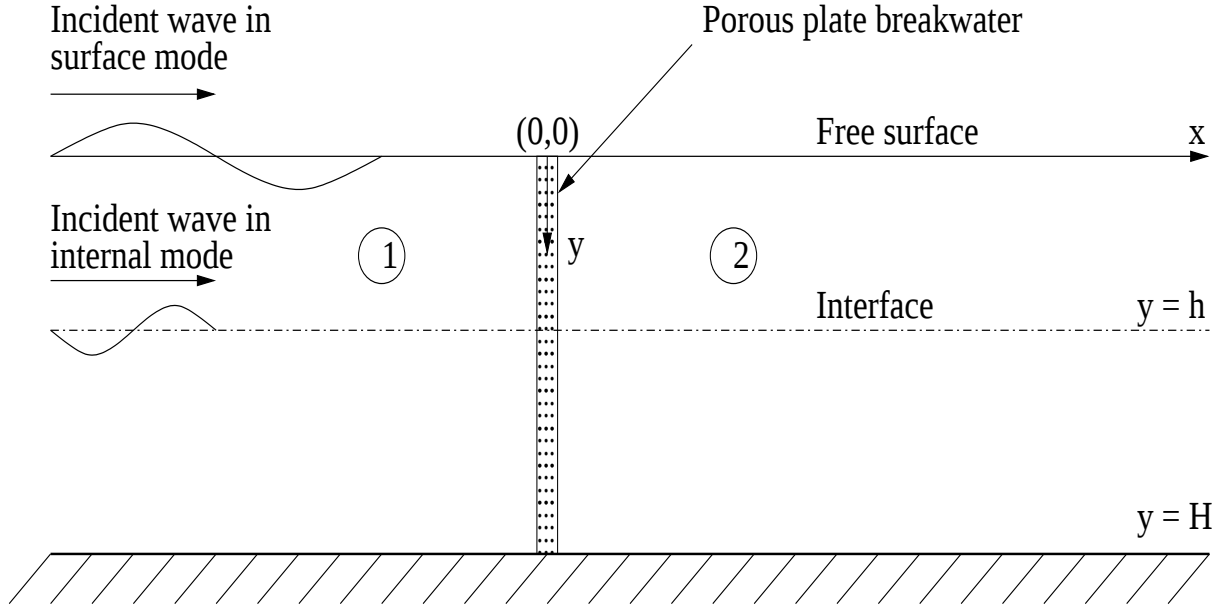


Figure 7.1: Definition sketch for flexible porous plate breakwater.

7.3 MODEL FOR FLUID FLOW

Similar to the case of porous membrane breakwater, by considering the waves incident from large negative x upon the flexible porous plate, the velocity potentials are obtained by eigenfunction-expansion method in each of the two regions 1 and 2 as marked in Fig. 7.1. The boundary value problem is defined by equation (3.1) along with the conditions Eqs. (3.12 – 3.14) and Eqs. 3.16 – 3.18). Applying the eigenfunction-expansion method, the velocity potentials in the regions 1 and 2 can be obtained and are same as given in Eqs. 6.1 and 6.2. Moreover the eigenfunctions f_n 's are also same as defined in Eq. 6.3.

As only open water regions exist in this case, the wave numbers p_n ($n = I, II$ for positive real roots and $n = 1, 2, 3, \dots$ for positive purely imaginary roots) are the roots of the dispersion relation in p as given in Eq. 4.8. The orthogonality condition as defined in the inner product Eq. 6.4 for the case of membrane is also applicable in the present problem. The reflection and transmission coefficients in SM and IM as defined in the case of membrane problem in Eq. 6.5 are also same in the present problem.

7.4 MODEL FOR FLEXIBLE PLATE RESPONSE

As described earlier in the general mathematical formulation, it is assumed that plate breakwater is deflected horizontally with displacement $\zeta(y, t) = \text{Re}[\xi(y)e^{-i\omega t}]$, where $\xi(y)$ represents the complex deflection amplitude and is assumed to be small as compared to the water depth. It is assumed that the plate breakwater is thin and behaves like a one-dimensional beam of uniform flexural rigidity EI and mass per unit length m_s . With these assumptions, the governing equation (Eq. 3.20) relating the plate displacement ξ from equilibrium to that of differential pressure acting on the plate at $x = 0$ can be applied. This gives

$$\frac{d^4\xi}{dy^4} - \beta^4\xi = \begin{cases} \frac{i\omega\rho_1}{EI}(\phi_2 - \phi_1), & \text{for } 0 < y < h, \\ \frac{i\omega\rho_2}{EI}(\phi_2 - \phi_1), & \text{for } h < y < H, \end{cases} \quad (7.1)$$

where β is the structural frequency parameter as defined by $\beta = (m_s\omega^2/EI)^{1/4}$. The breakwater will behave like a cantilever as it is assumed in the study that the breakwater has free and fixed ends at the free surface and seabed respectively.

The plate breakwater is clamped at the seabed and has a free end at the free surface. The corresponding boundary conditions as given in Eqs. 3.22 and 3.23 can be applied. This gives

$$\xi''(0) = 0, \quad \xi'''(0) = 0, \quad \xi(H) = 0, \quad \xi'(H) = 0. \quad (7.2)$$

The continuity condition across the interface (Eq. 3.29) should be imposed. Hence the deflection, slope of deflection, bending moment and the shear force acting on the plate breakwater are continuous at the interface (the point on the breakwater where the two fluids meet each other ($x = 0$; $y = h$)). This yield

$$\xi(h^-) = \xi(h^+), \quad \xi'(h^-) = \xi'(h^+), \quad \xi''(h^-) = \xi''(h^+), \quad \xi'''(h^-) = \xi'''(h^+). \quad (7.3)$$

Applying the porous boundary condition (Eq. 3.36) on the porous plate breakwater we obtain the same expression as defined in Eq. 6.9 and G is the complex porous-effect parameter as defined in Eq. 3.37.

7.5 GENERAL SOLUTION PROCEDURE

Applying the continuity of ϕ_x (Eq. 6.9) along the porous breakwater on $x = 0$ and invoking the orthogonality relation (Eq. 6.4) over $(0 < y < h) \cup (h < y < H)$, we can obtain the expression as defined in Eq. 6.10.

Utilizing the Eqs. 6.1, 6.2 and 6.10 a general solution is obtained for the fourth order non-homogeneous ODE, Eq. 7.1 (plate breakwater governing equation) and is given by

$$\xi(y) = E_1 e^{i\beta y} + E_2 e^{-i\beta y} + E_3 e^{\beta y} + E_4 e^{-\beta y} - \frac{2i\omega\rho}{EI} \sum_{n=I,II,1}^{\infty} \frac{R_n f_n(p_n, y)}{p_n^4 + \beta^4} \quad \text{for } (0 < y < H), \quad (7.4)$$

where the arbitrary constants E_i , $i = 1, 2, 3, 4$ and the fluid density ρ are defined as below.

$$E_i = \begin{cases} C_i & \text{for } 0 < y < h, \\ D_i & \text{for } h < y < H, \end{cases} \quad (i = 1, 2, 3, 4), \quad \rho = \begin{cases} \rho_1 & \text{for } 0 < y < h, \\ \rho_2 & \text{for } h < y < H. \end{cases} \quad (7.5)$$

Substituting the general solution for ξ (Eq. 7.4) in Eq. 6.9 and using the relations in Eqs. 6.1, 6.2 and 6.10 the following expression is derived.

$$h_0(y) - \sum_{n=I,II,1}^{\infty} R_n h_n(y) = 0, \quad (0 < y < H), \quad (7.6)$$

where

$$h_0(y) = \begin{cases} i[p_I I_I f_I(p_I, y) + p_{II} I_{II} f_{II}(p_{II}, y) - \omega(C_1 e^{i\beta y} + C_2 e^{-i\beta y} + C_3 e^{\beta y} + C_4 e^{-\beta y})], & \text{for } 0 < y < h, \\ i[p_I I_I f_I(p_I, y) + p_{II} I_{II} f_{II}(p_{II}, y) - \omega(D_1 e^{i\beta y} + D_2 e^{-i\beta y} + D_3 e^{\beta y} + D_4 e^{-\beta y})], & \text{for } h < y < H, \end{cases} \quad (7.7)$$

$$h_n(y) = \begin{cases} \left[\frac{2\omega^2 \rho_1}{(p_n^4 - \beta^4)EI} + ip_n + 2ip_I G \right] f_n(p_n, y), & \text{for } 0 < y < h, \\ \left[\frac{2\omega^2 \rho_2}{(p_n^4 - \beta^4)EI} + ip_n + 2ip_I G \right] f_n(p_n, y), & \text{for } h < y < H. \end{cases} \quad (n = I, II, 1, 2, \dots), \quad (7.8)$$

We can apply the least-squares-approximation method to Eq. 7.6 as described earlier in general mathematical formulation chapter. We can write

$$Q(y) = h_0(y) - \sum_{n=I,II,1}^N R_n h_n(y), \quad \text{for } 0 < y < H. \quad (7.9)$$

Applying the least-squares method, we obtain

$$\int_0^H \bar{\mathbf{Q}}(y) \frac{\partial Q(y)}{\partial R_n} dy = 0, \quad \text{for } n = I, II, 1, 2, \dots, N, \quad (7.10)$$

where the bar denotes the complex conjugate.

Eq. 7.10 provides $N + 2$ linear equations with $N + 10$ number of unknowns, as $h_0(y)$ involves 8 extra unknowns C_i 's and D_i 's (for $i = 1, 2, 3, 4$). Another required 8 linear equations are obtained from the breakwater end conditions (Eq. 7.2), interface conditions (Eq. 7.3) and the expression for ξ in Eq. 7.4. These system of equations are solved using Gauss-elimination method to compute and analyze various physical quantities of interest. Number of evanescent modes in the series are selected based on the experience of the numerical convergence experiment.

7.6 NUMERICAL RESULTS AND DISCUSSION

Numerical results are generated to study the combined effect of porosity and flexibility of breakwater on the wave motion in a two-layer fluid. The wave parameters are given in terms of the non-dimensional wave number $p_I H$, water depth h/H , fluid density ratio s and the breakwater parameters like the flexural rigidity $EI/\rho_2 g H^4$, porous-effect parameter G , and mass per unit length m_s . The breakwater mass m_s is kept fixed at $m_s = 10$ Kg/m² throughout the analysis (same numerical value for m_s is taken by Wang and Ren (1993)) because it is observed by Williams and Wang (2003) that the breakwater mass density has a minimal influence on the efficiency of the structure as a barrier to the wave motion.

A case study for the numerical convergence experiment is plotted in Fig. 7.2 (a and b), which depict the effect of number of selected evanescent modes, N on the accuracy of reflection/transmission coefficients in SM and IM respectively for wave past porous

plate breakwater. It may be seen from Fig. 7.2 (a and b), that for $N = 10$ and 15 the deviation in results is insignificant. In the present study, 15 evanescent modes are taken for computation of all numerical results.

7.6.1 Reflected and Transmitted Energy

In this subsection, the effects of various non-dimensional physical parameters on wave reflection and transmission in both SM and IM are analyzed. All results are presented with respect to the normalized SM wave number $p_I H$ by allowing normalized IM wave number $p_{II} H$ to vary, based on the dispersion relation. The effect of non-dimensional breakwater flexural rigidity on the reflection and transmission coefficients in SM and IM are shown in Fig. 7.3 (a) and (b) respectively. The general pattern of wave reflection in both SM and IM is that it increases from zero to its peak in the shallow water region and thereafter it reduces and there will be a negligible reflection in the deep water region. Similar observation are found for wave reflection by a plate barrier in a single-layer fluid in past (see, Figs. 4.4 – 4.8 of Williams and Wang (2003)). Wave reflection is increasing and wave transmission is reducing with the increase in the stiffness of the breakwater for both SM and IM wave motion. This is intuitively expected, as a stiffer structure will resist more waves, which as a result leads to higher wave reflection. Similar observations were made by Wang and Ren (1993) in the analysis of wave scattering by flexible barrier in a single-layer fluid domain of constant density.

The variation of reflection and transmission coefficients versus $p_I H$ in both the cases of SM and IM are plotted in Fig. 7.4 (a) and (b) respectively for different values of the porous-effect parameter G . Highest wave reflection and lowest wave transmission peaks (in both SM and IM cases) are observed for the case where breakwater has zero porosity, which is similar to the observations in case of a single-layer fluid by Williams and Wang (2003). However, it is observed that for G having large value of inertia effect of fluid inside the porous breakwater, wave reflection (in both SM and IM cases) is high. The probable reason may be that, in such situation the waves are obstructed significantly by the porous breakwater.

The effect of depth ratio h/H of two fluids on the reflection and transmission coefficients in SM and IM are shown in Fig. 7.5 (a) and (b) respectively. In SM wave motion, highest wave reflection peak is observed for $h/H = 0.25$ and lowest wave transmission peak is observed for $h/H = 0.5$ (see Fig. 7.5 (a)). However, the general pattern of wave reflection and transmission in case of SM wave motion is not significantly affected by the interface location. On the other hand, in IM wave motion, when the interface is located either very nearer to free surface ($h/H = 0.1$) or seabed ($h/H = 0.9$) (see Fig. 7.5 (b)), wave reflection increases and accordingly there is a reduction in the wave transmission. The probable reason for this change in general pattern of wave reflection and transmission in case of IM wave motion is due to the resonating interaction of surface- and internal-waves for $h/H = 0.1$ and it is due to influence of seabed for $h/H = 0.9$. In a two-layer fluid, the thin upper layer can be found in ocean where upper layer density changes due to solar heating and the thin lower layer can be found in ocean where near to the seabed the fluid density changes due to the mud and salinity.

The reflection and transmission coefficients are plotted versus $p_I H$ in SM and IM for various values of s in Fig. 7.6 (a) and (b) respectively. The general pattern of wave reflection and transmission in case of SM wave motion is not significantly affected by the change in the value of s (see Fig. 7.6 (a)). However, in case of wave motion in IM, when fluid density ratio approaches to 1 ($s = 0.99$ and 0.995) reflection coefficient increases initially with an increase in $p_I H$ and maintains a uniform value for higher wave number i.e., in the deep water region (see Fig. 7.6 (b)). In general, it is observed in case of IM wave motion that wave reflection is increasing and wave transmission is decreasing with the increase in the value of s . This is intuitively expected, as the fluid density ratio s approaches to unity, the interface elevation becomes significantly high (see Kundu and Cohen (2002) and Milne-Thomson (1996)). This increase in the elevation of the interface helps more waves in IM to reflect. For most of the situation in a real ocean, the fluid density ratio s is very close to one. This observation suggests that although the reflection coefficient for wave motion in SM is not significantly affected, the effect of the wave motion in IM cannot be neglected as $s \rightarrow 1$. For these kind of situations, the structure

will experience high wave load due to the impact of internal-waves.

7.6.2 Response of Plate Breakwater

In the present subsection, the breakwater response is analyzed. Unlike the case of reflection and transmission coefficients, the plate response is computed based on the combined effect of the waves in SM and IM apart from the local effects.

Variation of breakwater response $|\xi/I_T|$ for different values of non-dimensional breakwater flexural rigidity $EI/\rho_2 g H^4$, porous-effect parameter G and depth ratio h/H are plotted in Figs. 7.7 – 7.9. The breakwater deflection is increasing with a decrease in the rigidity of breakwater in Fig. 7.7. This is intuitively expected because less rigid structure will deform or bend more under the action of wave load. In Fig. 7.8, the breakwater displacement is high for $G = 0$. Similar observations are made in a single-layer fluid study (see, Wang and Ren (1993), Fig. 5). This is because less porous structure will experience a higher force. However, for very high porosity the strength of the structure will reduce and may cause higher bending (see the cases of $G = 2$ and $1 + 2i$). The bending of the breakwater is increasing with decrease in the value of h/H in Fig. 7.9. This is because, a cantilever will bend more when the location of the concentration of load is at higher distance from the fixed end. In addition, as the free surface and interface are close to each other, the wave load on the structure becomes high near the free surface.

7.6.3 Hydrodynamic Force on Plate Breakwater

In the subsection, the hydrodynamic force on the breakwater is analyzed. Unlike the case of reflection and transmission coefficients, hydrodynamic force is computed based on the combined effect of the waves in SM and IM apart from the local effects. The hydrodynamic force coefficient K_f is given by $K_f = |F_0/\rho_2 g H h|$, where

$$F_0 = i\omega \int_0^H \rho[\phi_2(0, y) - \phi_1(0, y)]dy. \quad (7.11)$$

Hydrodynamic force coefficients K_f acting on the breakwater versus non-dimensional flexural rigidity $EI/\rho_2 g H^4$ for different values of G and h/H are presented in Figs. 7.10

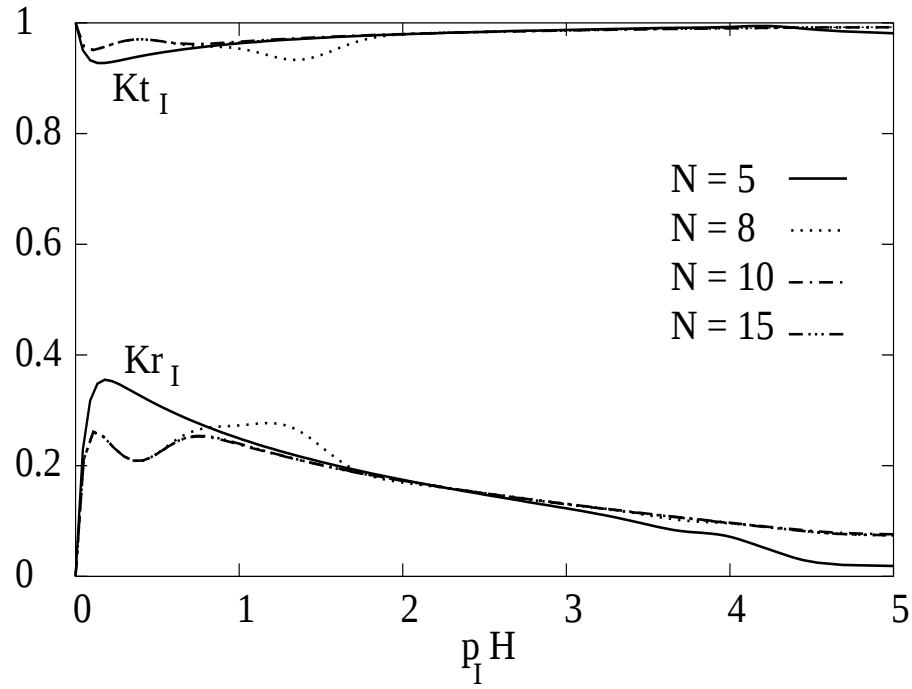
and 7.11. In general, the hydrodynamic force increases with an increase in flexural rigidity and then attains constant value for higher values of $EI/\rho_2 g H^4$. This is because at higher value of $EI/\rho_2 g H^4$, the breakwater start behaving like a rigid wall. Further, the hydrodynamic force on the structure reduces with an increase in porosity as expected (Fig. 7.10). Similar observations were made in case of a single-layer fluid by Wang and Ren (1993). The wave load on the breakwater is reducing with the increase in the value of h/H in Fig. 7.11. With an increase in the value of h/H , the combined effect of waves in SM and IM reduces and hence the force on the breakwater reduces.

7.6.4 Summary of Important Observations

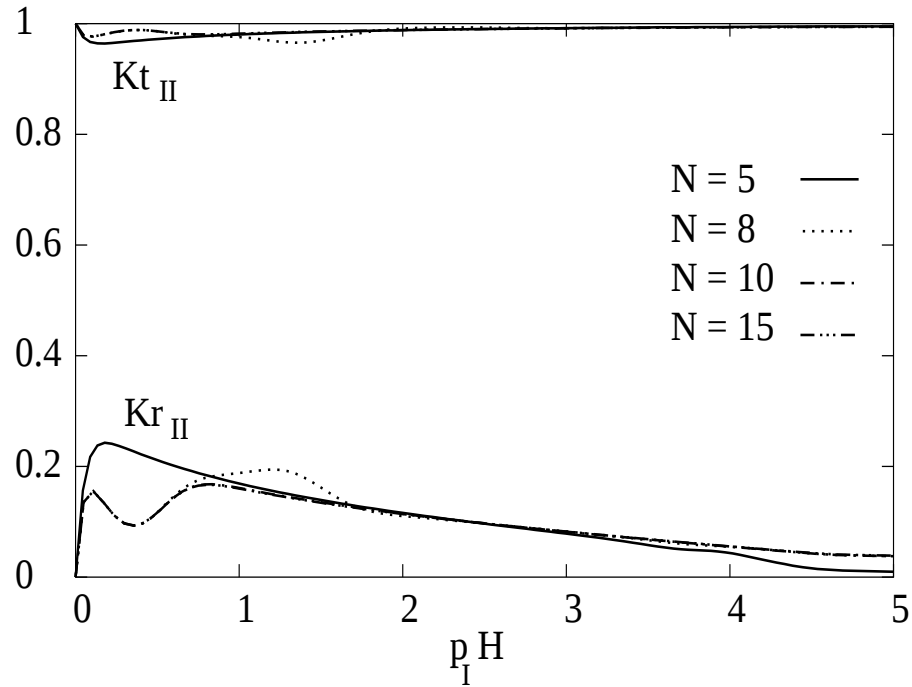
The important observations from the present numerical results for porous plate are summarized pointwise as below:

1. The general pattern of wave reflection in both SM and IM is that it increases from zero to it's peak in the shallow water region and there after it reduces and there will be a negligible reflection in the deep water region. Similar observations are found for wave reflection by a plate barrier in a single-layer fluid in past (see, Figs. 4.4 – 4.8 of Williams and Wang (2003)).
2. Wave reflection is increasing and wave transmission is reducing with the increase in the stiffness of the breakwater for both SM and IM wave motion. Similar observations were made by Wang and Ren (1993) in the analysis of wave scattering by flexible barrier in a single-layer fluid domain of constant density.
3. Highest wave reflection and lowest wave transmission peaks (in both SM and IM cases) are observed for the case where breakwater has zero porosity, which is similar to the observations in case of a single-layer fluid by Williams and Wang (2003).
4. Wave reflection (in both SM and IM cases) is high for G with large value of inertia effect of fluid inside the porous breakwater.
5. General pattern of wave reflection and transmission in case of SM wave motion is not significantly affected by the interface location.

6. In IM wave motion, when the interface is located either very near to free surface or seabed, wave reflection increases and accordingly there is a reduction in the wave transmission.
7. The general pattern of wave reflection and transmission in case of SM wave motion is not significantly affected by the change in the value of s .
8. In general, it is observed in case of IM wave motion that wave reflection is increasing and wave transmission is decreasing with the increase in the value of s .
9. Breakwater deflection is increasing with the decrease in the rigidity of breakwater.
10. Breakwater displacement is high for $G = 0$. Similar observations are made in a single-layer fluid study (see, Wang and Ren (1993), Fig. 5).
11. The bending of the breakwater is increasing with decrease in the value of h/H .
12. Hydrodynamic force increases with an increase in flexural rigidity and then attains constant value for higher values of $EI/\rho_2 g H^4$. Similar observations were made in case of a single-layer fluid by Wang and Ren (1993).
13. The wave load on the breakwater is reducing with the increase in the value of h/H .

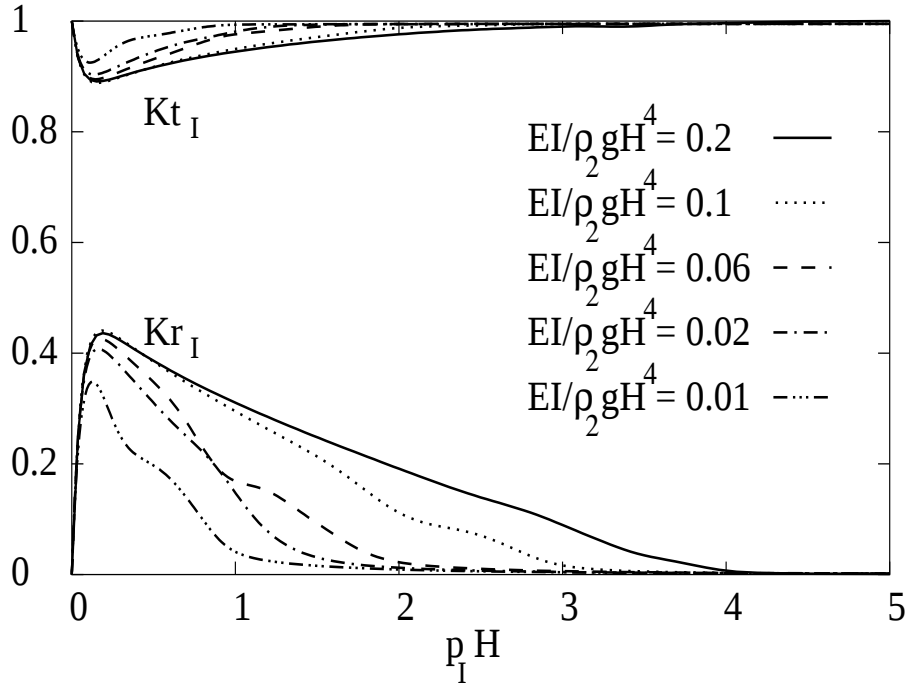


(a)

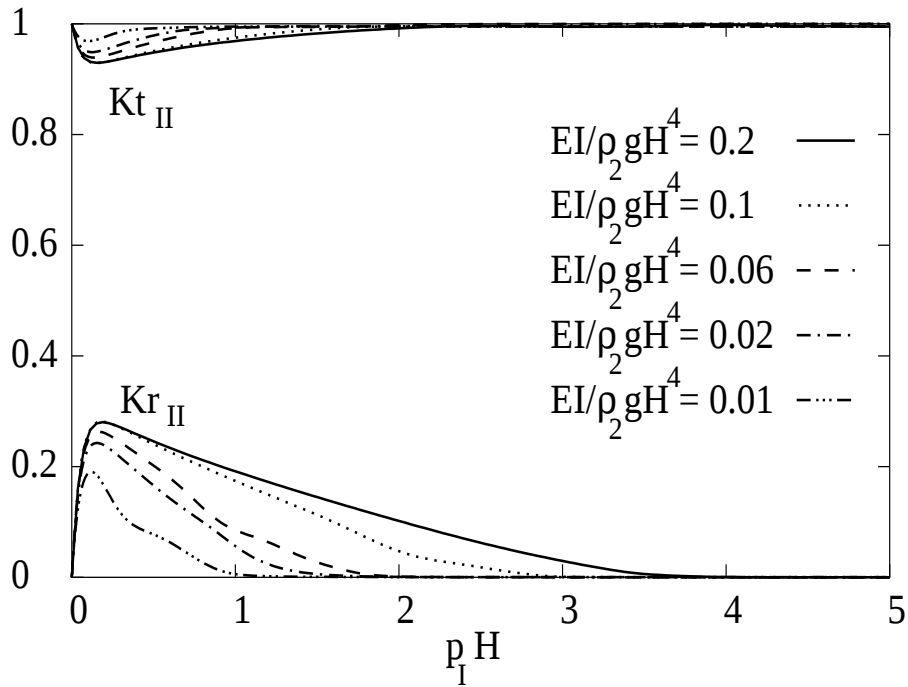


(b)

Figure 7.2: Convergence test for reflection and transmission coefficients in (a) SM and (b) IM versus $p_I H$ in case of wave past porous plate breakwater problem at $EI/\rho_2 g H^4 = 0.01$, $G = 1$, $s = 0.75$ and $h/H = 0.25$.

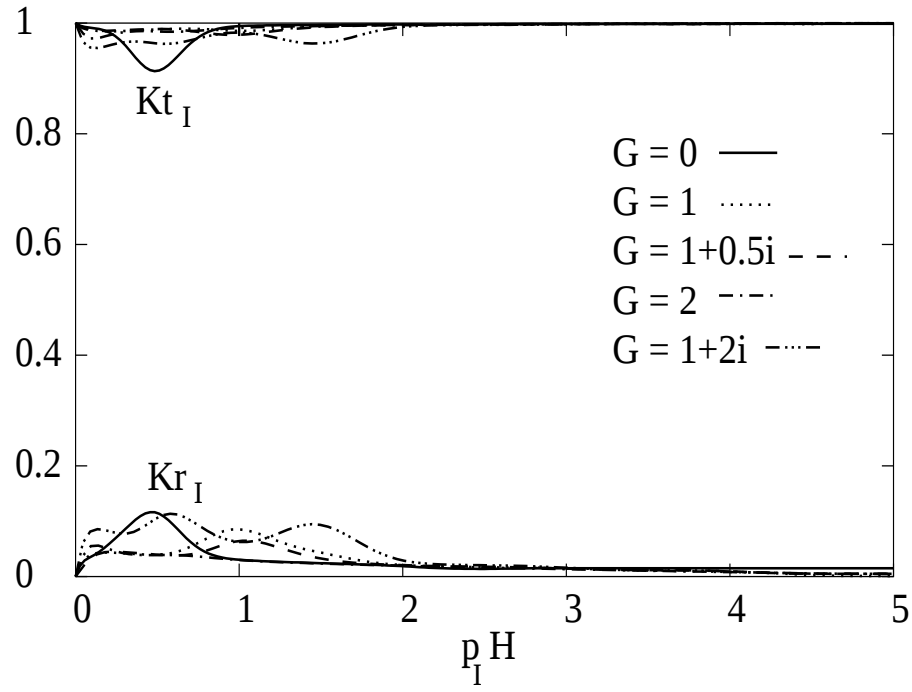


(a)

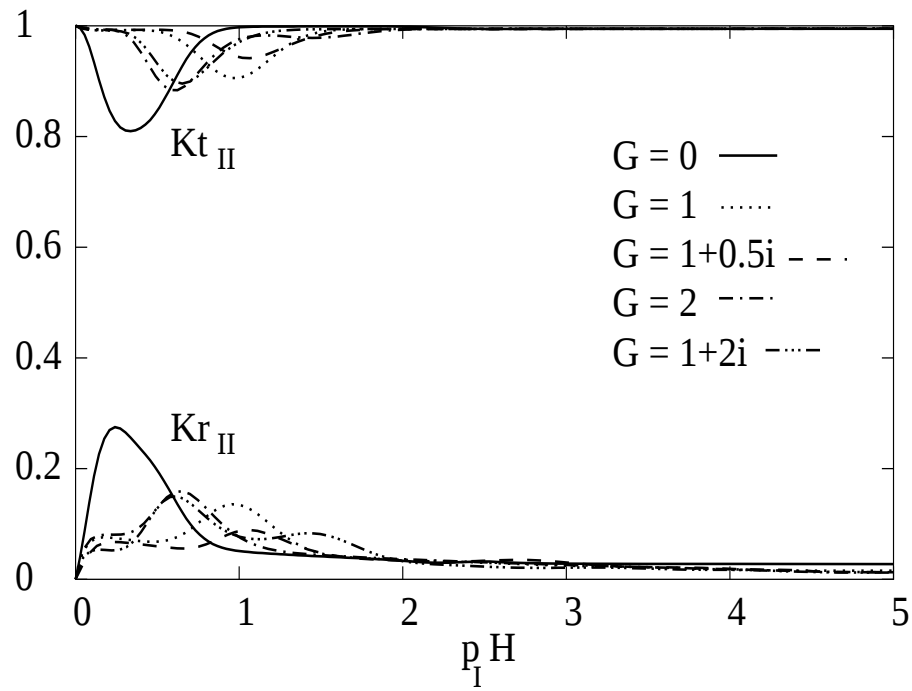


(b)

Figure 7.3: Reflection and transmission coefficients in (a) SM and (b) IM versus $p_I H$ for different $EI/\rho_2 g H^4$ values at $G = 1$, $s = 0.9$ and $h/H = 0.25$.

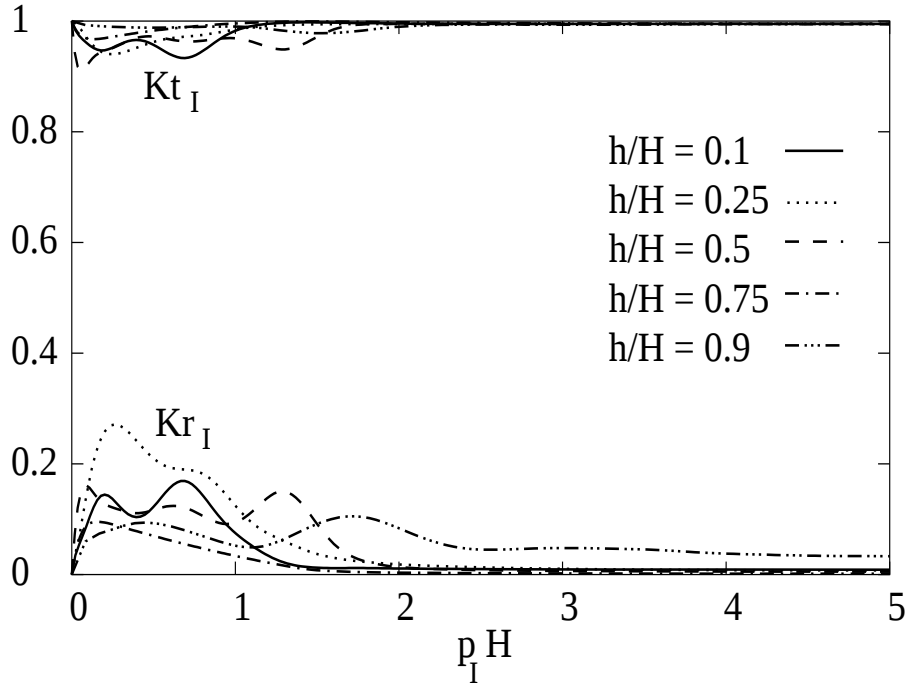


(a)

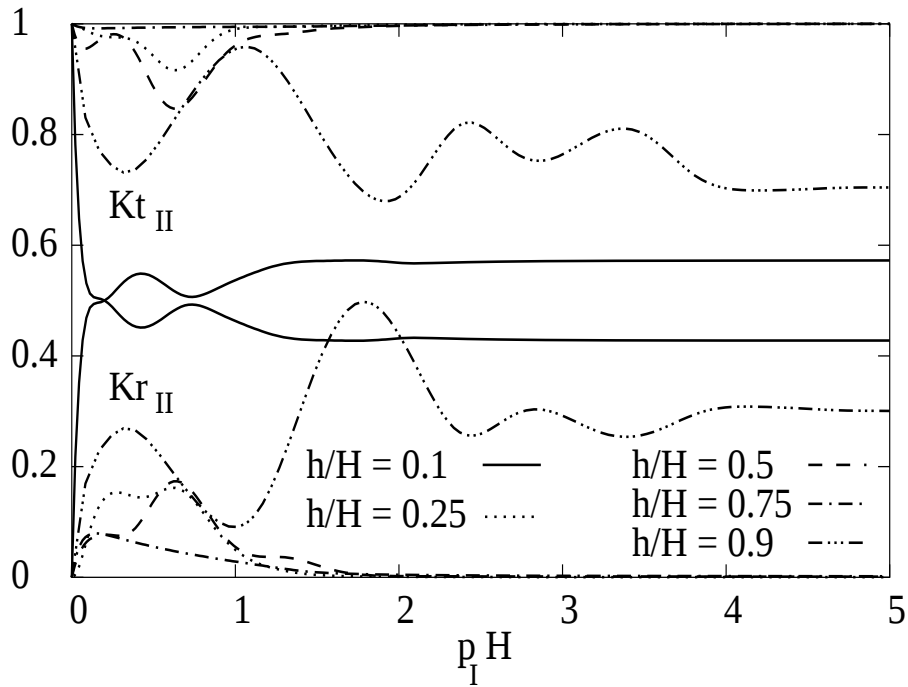


(b)

Figure 7.4: Reflection and transmission coefficients in (a) SM and (b) IM versus $p_I H$ for different G values at $h/H = 0.75$, $s = 0.75$ and $EI/\rho_2 g H^4 = 0.02$.

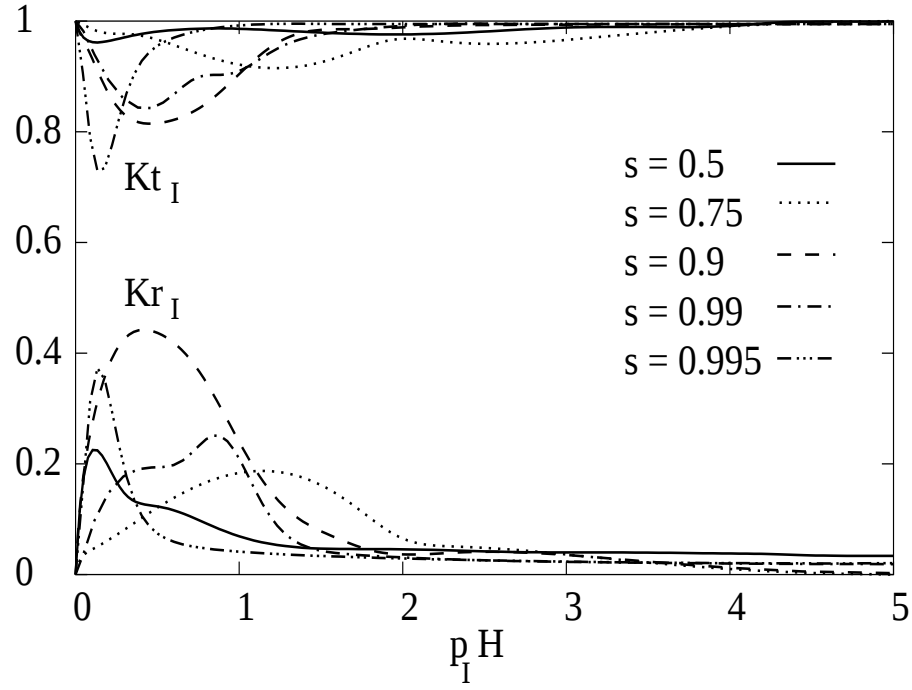


(a)

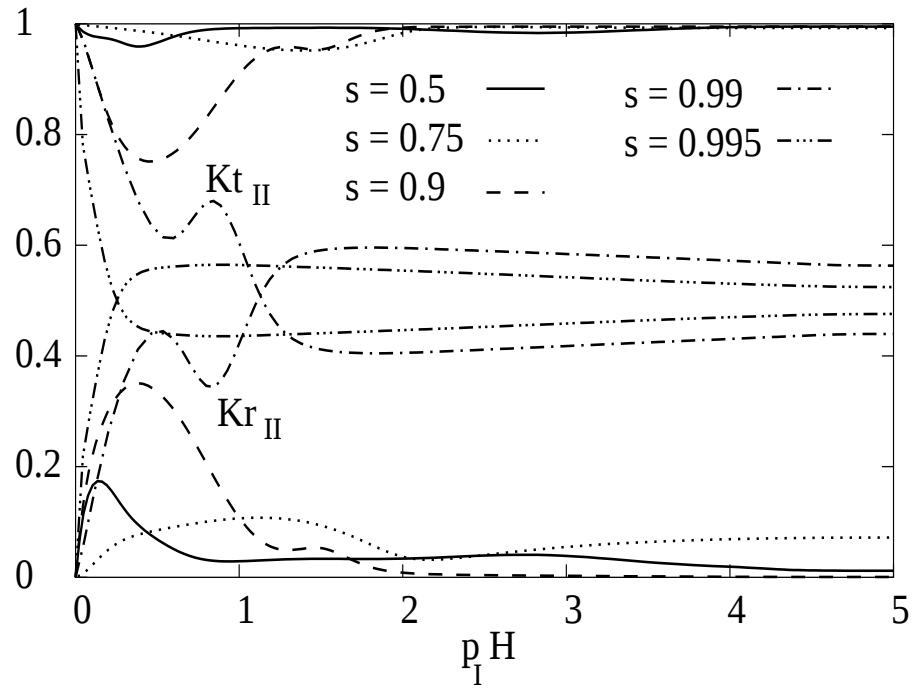


(b)

Figure 7.5: Reflection and transmission coefficients in (a) SM and (b) IM versus $p_I H$ for different h/H ratios at $G = 2$, $s = 0.9$ and $EI/\rho_2 g H^4 = 0.1$.



(a)



(b)

Figure 7.6: Reflection and transmission coefficients in (a) SM and (b) IM versus $p_I H$ for different s values at $h/H = 0.75$, $G = 1 + 0.5i$ and $EI/\rho_2 g H^4 = 0.06$.

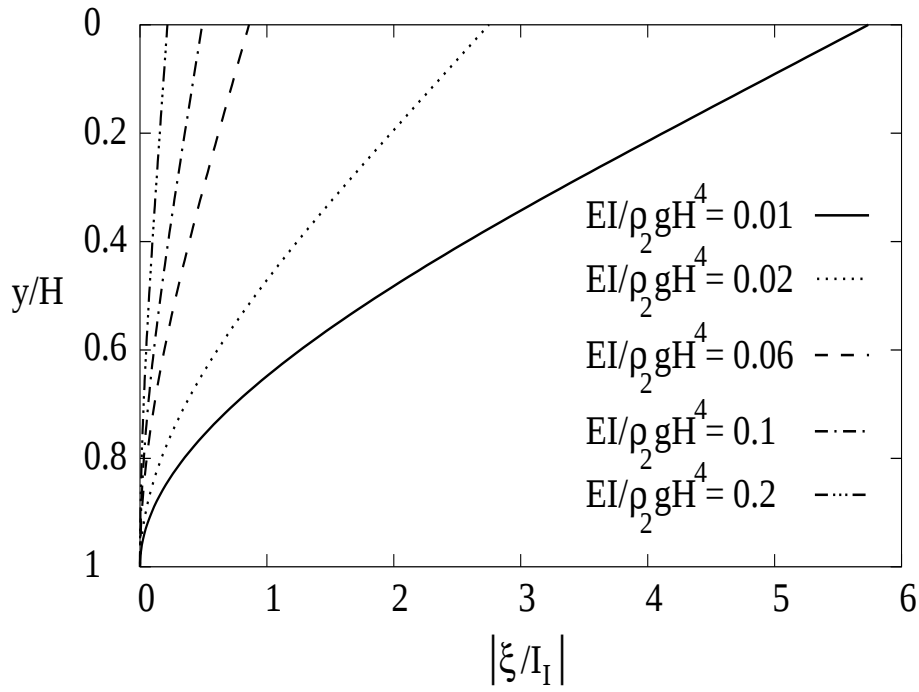


Figure 7.7: Breakwater displacement profile for different $EI/\rho_2 g H^4$ values at $h/H = 0.25$, $s = 0.9$, $p_I H = 0.5$ and $G = 1$.

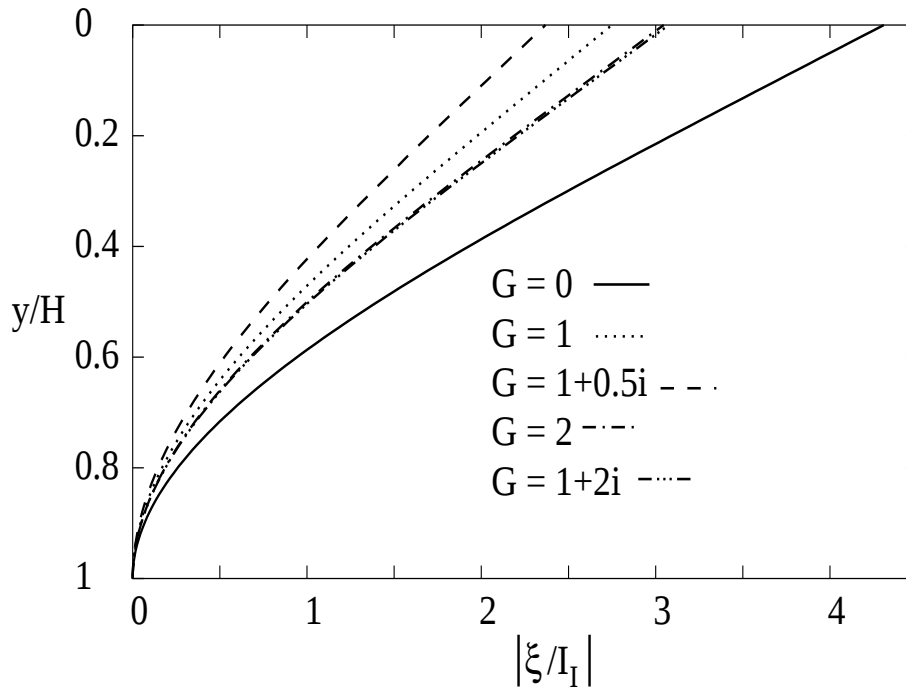


Figure 7.8: Breakwater displacement profile for different G values at $s = 0.9$, $p_I H = 0.5$, $h/H = 0.25$ and $EI/\rho_2 g H^4 = 0.02$.

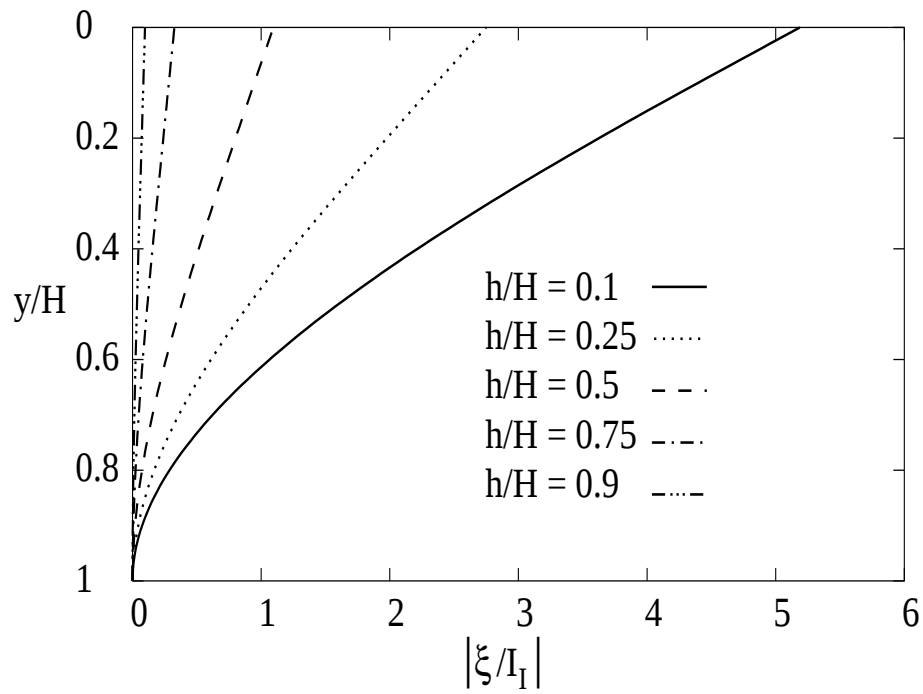


Figure 7.9: Breakwater displacement profile for different h/H ratios at $s = 0.9$, $p_I H = 0.5$, $EI/\rho_2 g H^4 = 0.02$ and $G = 1$.

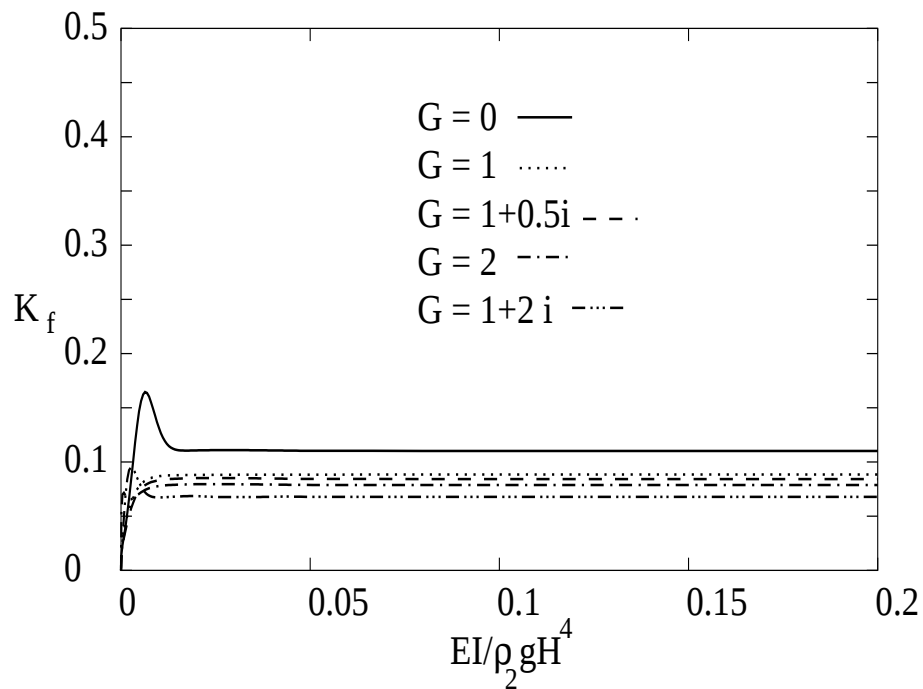


Figure 7.10: Force coefficient versus $EI/\rho_2 g H^4$ for different G values at $p_I H = 0.5$, $h/H = 0.25$, and $s = 0.9$.

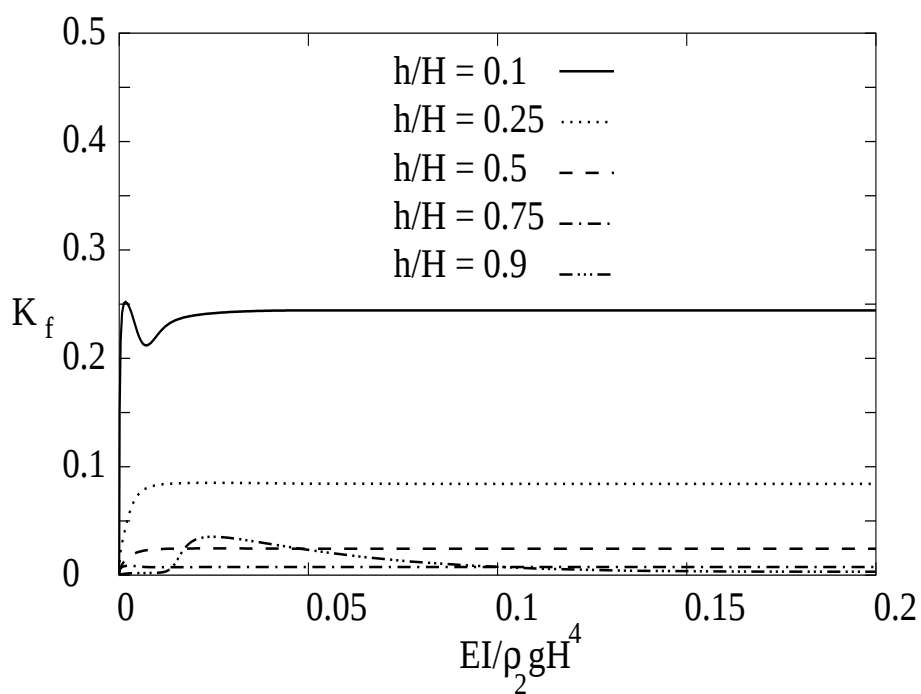


Figure 7.11: Force coefficient versus $EI/\rho_2 g H^4$ for different h/H ratios at $p_1 H = 0.5$, $s = 0.9$ and $G = 1 + 0.5i$.

Chapter 8

WAVE TRAPPING BY FLEXIBLE POROUS BREAKWATERS

8.1 INTRODUCTION

After analyzing wave scattering by flexible porous structures in the previous chapters, wave trapping by flexible porous partial breakwaters is considered in a two-layer fluid. Wave trapping in a two-layer fluid is a complex phenomena as it includes the trapping of surface- and internal-waves simultaneously. In the present chapter, the efficiency of a flexible porous partial plate breakwaters in trapping surface- and internal-waves near the end-wall of a semi-infinite long channel in a two-layer fluid domain is investigated based on the linearized-theory of water waves.

8.2 DEFINITION OF THE PHYSICAL PROBLEMS

In the present section, physical problems for wave trapping by flexible porous partial breakwaters in a two-layer fluid are considered. The two cases, bottom-standing and surface-piercing breakwaters, are illustrated in Figs. 8.1 and 8.2 respectively. In the two-layer fluid, the upper fluid has a free surface (undisturbed free surface located at $y = 0$) and the two fluids are separated by a common interface (undisturbed interface located

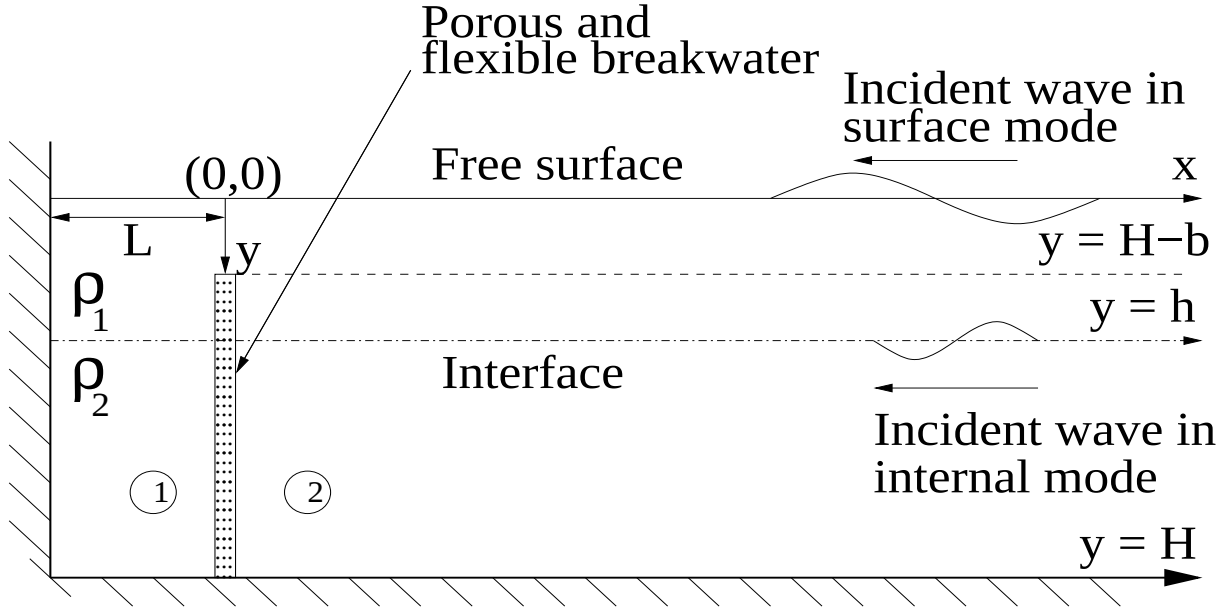


Figure 8.1: Definition sketch for wave trapping by bottom-standing partial plate breakwater.

at $y = h$), each fluid occupying the regions $-L < x < +\infty$; $0 < y < h$ in case of the upper fluid of density ρ_1 and $-L < x < +\infty$; $h < y < H$ in case of the lower fluid of density ρ_2 . L_{uf} and L_{lf} represent the notations for the portion of the breakwater which is in upper and lower fluid domain respectively where as notations L_g and L_{op} represent the regions, the gap and the portion of the breakwater which is above the free surface respectively. The porous breakwater is located at $x = 0$ with $L_{uf} = (H - b < y < h)$; $L_{lf} = (h < y < H)$ for a bottom-standing breakwater and $L_{op} = (-(h_1 - H) < y < 0)$; $L_{uf} = (0 < y < h)$ and $L_{lf} = (h < y < H - b)$ for a surface-piercing breakwater. On the other hand, the end-wall is located at $x = -L$; $0 < y < H$ (see Figs. 8.1 and 8.2).

8.3 MODEL FOR FLUID FLOW

The fluid velocity potential will satisfy the condition (3.15) on the impermeable end-wall.

Hence

$$\frac{\partial \phi}{\partial x} = 0 \text{ on } x = -L, \quad 0 < y < H. \quad (8.1)$$

Considering the waves incident from large positive x upon the flexible porous partial breakwaters, the velocity potentials are obtained by eigenfunction-expansion method sim-

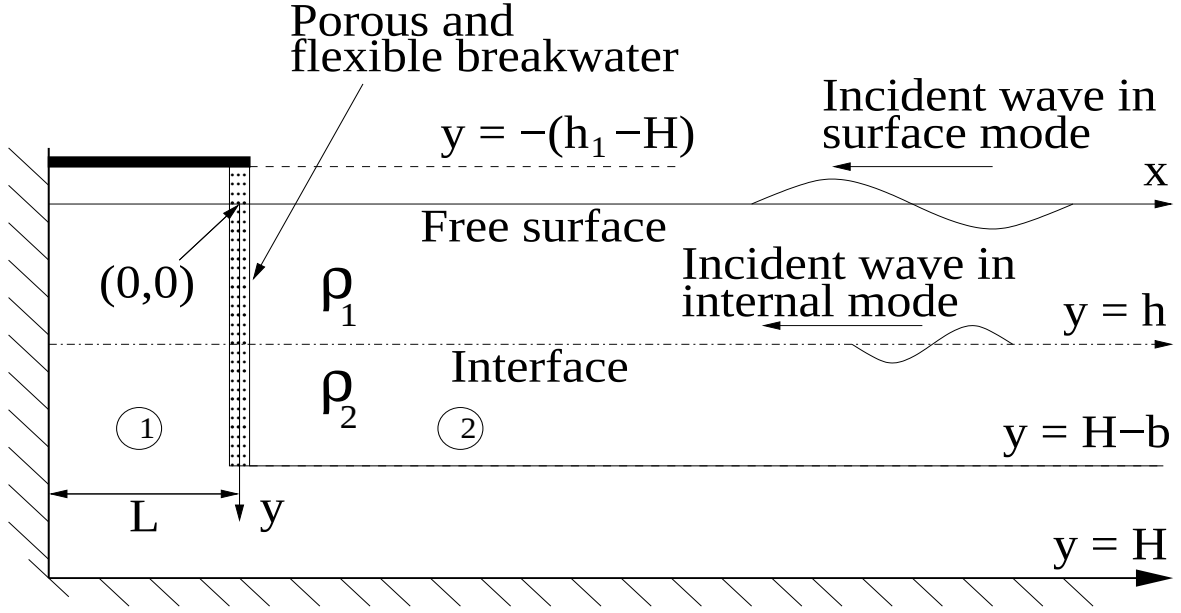


Figure 8.2: Definition sketch for wave trapping by surface-piercing partial plate breakwater.

ilar to the case of a surface-piercing dike, in each of the two regions 1 and 2 as marked in Figs. 8.1 and 8.2. The present fluid problem is also a boundary value problem which satisfies the fluid flow governing equation (3.1) along with the conditions Eqs. (3.12 – 3.14), (3.16 – 3.17) and (8.1). Applying the eigenfunction-expansion method, the velocity potentials in the regions 1 and 2 are obtained and are given by

$$\phi_1 = \sum_{n=I,II,1}^{\infty} A_n \cos p_n(L+x) f_n(p_n, y), \text{ for } -L < x < 0, \quad (8.2)$$

and

$$\phi_2 = \sum_{n=I}^{II} I_n e^{-ip_n x} f_n(p_n, y) + \sum_{n=I,II,1}^{\infty} R_n e^{ip_n x} f_n(p_n, y), \text{ for } x > 0, \quad (8.3)$$

The eigenfunctions f_n 's are again same as defined in Eq. 6.3 and A_n, R_n ($n = I, II, 1, 2, \dots$) are unknown constants to be determined.

In this case also only open water regions exist like in the previous two cases. Thus, the wave numbers p_n ($n = I, II$ for positive real roots and $n = 1, 2, 3, \dots$ for positive purely imaginary roots) are the roots of the dispersion relation in p as given in Eq. 4.8.

The orthogonality condition as defined in the inner product Eq. 6.4 for the case of membrane problem is also applicable in the present problem. The reflection coefficients in

SM and IM are defined in the case of membrane problem in Eq. 6.5 is also applicable in the present problem. It may be noted that in the present case there are no transmission coefficients because of the presence of the impermeable end-wall.

8.4 MODEL FOR FLEXIBLE PLATE RESPONSE

As described earlier in the general mathematical formulation, it is assumed that plate breakwater is deflected horizontally with displacement $\zeta(y, t) = \text{Re}[\xi(y)e^{-i\omega t}]$, where $\xi(y)$ represents the complex deflection amplitude and is assumed to be small as compared to the water depth. It is assumed that the plate breakwater is thin and behaves like a one-dimensional beam of uniform flexural rigidity EI and mass per unit length m_s . With these assumptions, the governing equation (Eq. 3.20) relating the plate displacement ξ from equilibrium to that of differential pressure acting on the plate at $x = 0$ can be applied. This gives

$$\frac{d^4\xi}{dy^4} - \beta^4\xi = \begin{cases} 0, & \text{on } y \in L_{op}, \\ \frac{i\omega\rho_1(\phi_1 - \phi_2)}{EI}, & \text{on } y \in L_{uf}, \\ \frac{i\omega\rho_2(\phi_1 - \phi_2)}{EI}, & \text{on } y \in L_{lf}, \end{cases} \quad (8.4)$$

where β is the structural frequency parameter as defined by $\beta = (m_s\omega^2/EI)^{1/4}$. The breakwater will behave like a cantilever as it is assumed in the present study that the bottom-standing breakwater is fixed at the seabed and is having a free edge inside the fluid domain, and on the other hand, the surface-piercing breakwater is clamped above the free surface and the free end is immersed inside the fluid domain.

The bottom-standing plate breakwater (Fig. 8.1) is fixed at the seabed and is having a free edge inside the fluid domain. The corresponding boundary conditions as given in Eqs. 3.22 and 3.23 can be applied. This gives

$$\xi(H) = 0, \quad \xi'(H) = 0, \quad \xi''(H - b) = 0 \text{ and } (\xi''')(H - b) = 0. \quad (8.5)$$

On the other hand, the surface-piercing breakwater (Fig. 8.2) is clamped above the free surface and the free end is immersed inside the fluid domain. The corresponding boundary

conditions as given in Eqs. 3.22 and 3.23 can be applied. This gives

$$\xi(-(h_1 - H)) = 0, \quad \xi'(-(h_1 - H)) = 0, \quad \xi''(H - b) = 0 \text{ and } (\xi''')(H - b) = 0. \quad (8.6)$$

The continuity condition across the interface (Eq. 3.29) should be imposed. Hence the deflection, slope of deflection, bending moment and the shear force acting on the plate breakwater are continuous at the interface (the point on the breakwater where the two fluids meet each other ($x = 0$; $y = h$)). This yield

$$\xi(h^-) = \xi(h^+), \quad \xi'(h^-) = \xi'(h^+), \quad \xi''(h^-) = \xi''(h^+), \quad \xi'''(h^-) = \xi'''(h^+). \quad (8.7)$$

In case of surface-piercing breakwater continuity condition across the free surface (Eq. 3.29) should be imposed. Hence, similar to the case of interface, the deflection, slope of deflection, bending moment and the shear force acting on the plate breakwater are continuous at the free surface (the point on the breakwater where the air and the upper fluid meet each other ($x = 0$; $y = 0$)). This yield

$$\xi(0^-) = \xi(0^+), \quad \xi'(0^-) = \xi'(0^+), \quad \xi''(0^-) = \xi''(0^+), \quad \xi'''(0^-) = \xi'''(0^+). \quad (8.8)$$

Applying the porous boundary condition (Eq. 3.36) on the porous plate breakwater we obtain

$$\frac{\partial \phi_j}{\partial x} = ik_0 G(\phi_2 - \phi_1) + i\omega \xi \quad (j = 1, 2) \text{ on } x = 0, 0 < y < H. \quad (8.9)$$

and G is the complex porous-effect parameter as defined in Eq. 3.37.

The condition across the gap Eq. 3.19 must be imposed. This gives

$$\phi_1 = \phi_2, \text{ and } \frac{\partial \phi_1}{\partial x} = \frac{\partial \phi_2}{\partial x} \quad \text{on } x = 0, y \in L_g. \quad (8.10)$$

The conditions Eq. 8.9 and Eq. 8.10 can be rewritten as

$$ik_0 G(\phi_2 - \phi_1) = \begin{cases} 0 & \text{on } x = 0, y \in L_g, \\ \frac{\partial \phi_j}{\partial x} - i\omega \xi & \text{for } j = 1, 2 \text{ on } x = 0, y \in L_{uf} + L_{lf}. \end{cases} \quad (8.11)$$

8.5 GENERAL SOLUTION PROCEDURE

Applying the continuity of ϕ_x (Eqs. 8.9 and 8.10) along the porous breakwater for $y \in L_{uf} + L_{lf}$ on $x = 0$ and the gap $y \in L_g$ on $x = 0$, and invoking the orthogonality relation (Eq. 6.4) over $(0 < y < h) \cup (h < y < H)$, we obtain

$$I_n - R_n = -A_n \sin p_n L \quad (n = I, II), \quad R_n = -A_n \sinh p_n L \quad (n = 1, 2, \dots). \quad (8.12)$$

From Eq. 8.12 it is clear that the reflection coefficient $Kr_n = \left| \frac{R_n}{I_n} \right| = 1$ when $L = j\lambda_n/2$ ($n = I, II$ and $j = 0, 1, 2, \dots$). It suggests that when the distance between the end-wall and the breakwater is an integer multiple of half wavelength of the incident wave in a particular wave mode (surface- or internal-wave mode), maximum reflection takes place in the corresponding wave mode irrespective of breakwater configurations (bottom-standing/surface-piercing, flexible/rigid, impermeable/permeable breakwater).

Utilizing the Eqs. 8.2, 8.3 and 8.12 a general solution is obtained for the fourth order non-homogeneous ODE, Eq. 8.4 (partial plate breakwater governing equation) and is given by

$$\xi(y) = E_1 e^{i\beta y} + E_2 e^{-i\beta y} + E_3 e^{\beta y} + E_4 e^{-\beta y} + \sum_{n=I}^{II} H_n I_0 f_n(p_n, y) + \sum_{n=I, II, 1}^{\infty} \bar{H}_n R_n f_n(p_n, y), \quad (8.13)$$

where bar denotes the complex conjugate, the arbitrary constants E_i , $i = 1, 2, 3, 4$ and H_n are defined as below.

$$E_i = \begin{cases} E_{op_i} & \text{for } y \in L_{op}, \\ E_{uf_i} & \text{for } y \in L_{uf}, \\ E_{lf_i} & \text{for } y \in L_{lf}, \end{cases} \quad (8.14)$$

and

$$H_n = 0, \quad y \in L_{op} \quad \text{and} \quad H_n = \frac{i\omega\rho_j(1 - i\cot p_n L)}{EI(p_n^4 - \beta^4)}, \quad 0 < y < H, \quad (8.15)$$

and

$$\rho_j = \begin{cases} 0 & \text{for } y \in L_{op}, \\ \rho_1 & \text{for } y \in L_{uf}, \\ \rho_2 & \text{for } y \in L_{lf}. \end{cases} \quad (8.16)$$

Eop_i , Euf_i , Elf_i for $i = 1, 2, 3, 4$ are unknown constants to be determined. Substituting ϕ_1 , ϕ_2 and ξ from Eqs. 8.2, 8.3 and 8.13 in Eq. 8.11 and utilizing relation (8.12), we obtain

$$g_0(y) + \sum_{n=I,II,1}^{\infty} g_n(y)R_n = 0 \quad \text{for } 0 < y < H, \quad (8.17)$$

where

$$g_0(y) = \begin{cases} \sum_{n=I}^{II} I_n r_n(y) + i\omega Euf_1 e^{i\beta y} + i\omega Euf_2 e^{-i\beta y} + i\omega Euf_3 e^{\beta y} + \\ \quad i\omega Euf_4 e^{-\beta y} & \text{for } 0 < y < h, \\ \sum_{n=I}^{II} I_n r_n(y) + i\omega Elf_1 e^{i\beta y} + i\omega Elf_2 e^{-i\beta y} + i\omega Elf_3 e^{\beta y} + \\ \quad i\omega Elf_4 e^{-\beta y} & \text{for } h < y < H, \end{cases} \quad (8.18)$$

$$g_n(y) = \begin{cases} (GP_n + UB_n)f_n(p_n, y) & \text{for } 0 < y < h, \\ & n = (I, II, 1, 2, \dots) \\ (GP_n + LB_n)f_n(p_n, y) & \text{for } h < y < H, \end{cases} \quad (8.19)$$

$$r_n(y) = \begin{cases} (\bar{G}P_n + ZUB_n)f_n(p_n, y) & \text{for } 0 < y < h, \\ & n = (I, II) \\ (\bar{G}P_n + ZLB_n)f_n(p_n, y) & \text{for } h < y < H. \end{cases} \quad (8.20)$$

The definition of the aforementioned notations and the range of their validity are as given below

$$GP_n = 1 + i\cot p_n L \quad \text{for } y \in L_g, \quad (8.21)$$

$$UB_n = iGk_0 GP_n + ip_n - \frac{\omega^2 \rho_1 GP_n}{EI(p_n^4 - \beta^4)} \quad \text{for } y \in L_{uf}, \quad (8.22)$$

$$LB_n = iGk_0 GP_n + ip_n - \frac{\omega^2 \rho_2 GP_n}{EI(p_n^4 - \beta^4)} \quad \text{for } y \in L_{lf}, \quad (8.23)$$

$$ZUB_n = iGk_0\bar{G}P_n - ip_n - \frac{\omega^2\rho_1\bar{G}P_n}{EI(p_n^4 - \beta^4)} \text{ for } y \in L_{uf}, \quad (8.24)$$

and

$$ZLB_n = iGk_0\bar{G}P_n - ip_n - \frac{\omega^2\rho_2\bar{G}P_n}{EI(p_n^4 - \beta^4)} \text{ for } y \in L_{lf}. \quad (8.25)$$

We can apply the least-squares-approximation method to Eq. 8.17 as described earlier in general mathematical formulation chapter. We can write

$$Q(y) = g_0(y) + \sum_{n=I,II,1}^N R_n g_n(y), \text{ for } 0 < y < H. \quad (8.26)$$

Applying the least-squares method, we obtain

$$\int_0^H \bar{Q}(y) \frac{\partial Q(y)}{\partial R_n} dy = 0, \text{ for } n = I, II, 1, 2, \dots, N, \quad (8.27)$$

where the bar denotes the complex conjugate.

Eq. 8.27 provides $N + 2$ linear equations with $N + 10$ unknowns in case of bottom-standing breakwater and $N + 14$ unknowns in case of surface-piercing breakwater, as $h_0(y)$ involves extra unknowns Eop_i , Euf_i , Elf_i for $i = 1, 2, 3, 4$. In case of bottom-standing breakwater 8 more equations are needed to solve the matrix systems and the required another 8 linear equations are obtained by substituting the expression for ξ from Eq. 8.13 in the breakwater end conditions Eq. 8.5 and interface continuity conditions Eq. 8.7. On the other hand, in case of surface-piercing breakwater the required 12 more equations are obtained by substituting the expression for ξ from Eq. 8.13 in the breakwater end conditions Eq. 8.6, free surface and interface continuity conditions Eqs. 8.8 and 8.7. These system of equations are solved using Gauss-elimination method to compute and analyze various physical quantities of interest. Number of evanescent modes in the series are selected based on the experience of the numerical convergence experiment.

8.6 NUMERICAL RESULTS AND DISCUSSION

In general, the behavior of reflected wave energy is one of the major criteria in deciding the effectiveness of a breakwater in trapping the water waves. It is well known in the literature that trapping occurs between a breakwater and the end-wall for specific wave

frequencies, which leads to minimum wave reflection by the breakwater. In the present study, numerical results are computed to investigate the performance of both bottom-standing and surface-piercing partial breakwaters in trapping the surface- and internal-waves in a two-layer fluid by keeping the mass per unit length m_s of the breakwater fixed at 10 Kg/m². The influence of various physical parameters like water depth h/H , fluid density ratio s , porous-effect parameter G of the breakwater, length of submergence of breakwater $(H - b)/H$, total length of breakwater h_1/H in case of surface-piercing breakwater, breakwater flexural rigidity EI on wave trapping in both SM and IM wave motion, and the nature of hydrodynamic force Fr and breakwater deflection ξ are studied. The definition of hydrodynamic force Fr is given by

$$Fr = i\omega \int_{L_{uf}+L_{lf}} \rho_j [\phi_1(0, y) - \phi_2(0, y)] dy. \quad (8.28)$$

It is observed that in both the cases of bottom-standing and surface-piercing breakwaters reflection coefficients in SM, Kr_I and in IM, Kr_{II} are periodic in nature for normalized breakwater positions L/λ_I and L/λ_{II} respectively. Full reflection takes place at $L/\lambda_j = n/2$, for $n = 1, 2, \dots, j = I, II$ and minimum reflection occurs at an intermediate points of $(n - 1)/2 < L/\lambda_j < n/2$, for $n = 1, 2, \dots$ and $(j = I, II)$. A similar phenomenon of minimum reflection in case of single-layer fluid is referred as wave trapping in the literature (see Sahoo et al. (2000) and Yip et al. (2002)).

8.6.1 The Case of a Bottom-Standing Breakwater

The influence of various important physical parameters on trapping of both surface- and internal-waves by bottom-standing partial breakwaters is discussed in this subsection.

Wave Reflection

Fig. 8.3 (a) and (b) depict the effect of bottom-standing breakwater stiffness EI on reflection coefficients Kr_I in SM and Kr_{II} in IM respectively. In Fig. 8.3 (a), minimum wave reflection and hence high wave trapping is observed in the case of waves in SM for higher values of EI . Since smaller values of EI enhances wave transmission across the

breakwater, wave trapping is observed to be negligible. However, the difference in the reflection coefficients for large EI values ($EI = 40, 100 \text{ Nm}$) is marginal. It is due to the fact that after certain rise in the value of flexural rigidity, the breakwater starts behaving like a rigid wall and the reflection coefficient is not affected with further rise in breakwater rigidity. On the other hand, in the case of wave motion in IM the influence of change in breakwater flexural rigidity is insignificant (Fig. 8.3 (b)).

The variation of reflection coefficients versus L/λ_j for ($j = I, II$) in both the cases of SM and IM are plotted in Fig. 8.4 (a) and (b) respectively for different values of the porous-effect parameter G . In Fig. 8.4 (a), the amount of trapped waves in SM is found to be increasing with a decrease in the porous-effect parameter $|G|$. On the other hand, in the case of wave motion in IM, the amount of trapped waves is found to be less for both small and large value of $|G|$ (Fig. 8.4 (b)). Highest wave trapping in IM is observed for moderate values of porous-effect parameters $G = 1 + 1i$ and $G = 2$. Interestingly, for both SM and IM, wave motion at $G = 1 + 1i$ and $G = 2$, the minimum value of the reflection coefficient is same in magnitude and the only difference is the breakwater location at which the minimum reflection coefficient is observed. This may be due to the change in phase angle of the porous-effect parameter.

The influence of non-dimensional breakwater length b/H of the bottom-standing breakwater on the reflection coefficients in SM and IM are shown in Fig. 8.5 (a) and (b) respectively. In Fig. 8.5 (a) for SM wave motion, highest wave trapping is observed when the breakwater length, b/H is just exceeds the depth of the lower fluid, $(1 - h/H)$ (at $b/H = 0.6$). On the other hand, the change in breakwater length has negligible effect to trap the waves in IM in the region between the breakwater and the channel end-wall (Fig. 8.5 (b)).

The influence of interface location h/H on the reflection coefficients in SM and IM are shown in Fig. 8.6 (a) and (b) respectively. Highest wave trapping is observed in SM wave motion when interface is nearer to the seabed. However, for $h/H = 0.25$ and 0.5 the reflection coefficients are found to be same (Fig. 8.6 (a)). On the other hand, highest wave trapping is observed in IM wave motion when the interface is located just at the

center of free surface and the seabed (Fig. 8.6 (b)).

The reflection coefficients versus L/λ_j for ($j = I, II$) are plotted in SM and IM for various values of s in Fig. 8.7 (a) and (b) respectively. In Fig. 8.7 (a) wave trapping in SM is found to be increasing with the increase in the value of s . On the other hand, the reflection coefficient in IM follows an opposite trend compared to the case of SM wave motion and the wave trapping is found to be decreasing with the increase in the value of s (Fig. 8.7 (b)).

Breakwater Deflection and Hydrodynamic Force

The bottom-standing breakwater deflection and hydrodynamic forces are plotted in Figs. 8.8 and 8.9. Except for $b/H = 0.8$, in general the breakwater deflection decreases with the decrease in the value of b/H (Fig. 8.8 (a)) and in Fig. 8.8 (b), breakwater deflection increases with the decrease in the value of h/H . In Fig. 8.9 (a), the hydrodynamic force is decreasing with the increase in the value of G . However, higher hydrodynamic forces are observed for G with higher values of inertial effect of fluid inside the porous media, which is represented by the imaginary part of G . On the other hand, the hydrodynamic force is increasing with the increase in the value of s (Fig. 8.9 (b)), which is due to the high inertial wave amplitude as s approaches to one (see Kundu and Cohen (2002)).

8.6.2 The Case of a Surface-Piercing Breakwater

The influence of various important physical parameters on trapping of both surface- and internal-waves by surface-piercing partial breakwaters is discussed in this subsection.

Wave Reflection

Fig. 8.10 (a) and (b) depict the effect of surface-piercing breakwater stiffness EI on reflection coefficients Kr_I in SM and Kr_{II} in IM respectively. In Fig. 8.10 (a) minimum wave reflection and hence high wave trapping is observed for moderate value of breakwater flexural rigidity ($EI = 40$ Nm). On the other hand, in the case of wave motion in IM

the maximum wave trapping is observed for high value of breakwater flexural rigidity ($EI = 100 \text{ Nm}$) (Fig. 8.10 (b)).

The variation of reflection coefficients versus normalized breakwater position L/λ_j for ($j = I, II$) in both the cases of SM and IM are plotted in Fig. 8.11 (a) and (b) respectively for different values of the porous-effect parameter G . In Fig. 8.11 (a), the amount of trapped waves in SM is found to be increasing with a decrease in the porous-effect parameter $|G|$. On the other hand, in the case of wave motion in IM the amount of trapped waves is found to be less for both small and large value of $|G|$ (Fig. 8.11 (b)).

The influence of non-dimensional length of submergence $1 - b/H$ in the case of surface-piercing breakwater on wave trapping in both SM and IM are shown in Fig. 8.12 (a) and (b) respectively. In SM wave motion, the amount of wave energy trapped is found to be increasing with an increase in $1 - b/H$. However, the numerical value of minimum of the reflection coefficients for $1 - b/H = 0.6$ and 0.7 are same. This is because most of the wave energy in SM concentrated in the upper fluid and this wave energy decays exponentially towards the seabed (Fig. 8.12 (a)). On the other hand, wave trapping in IM is found to be high when the length of submergence of the breakwater is equal or greater than the depth of upper fluid (Fig. 8.12 (b)).

The effect of change in non-dimensional length h_1/H of the surface-piercing breakwater on the reflection coefficients in SM and IM are shown in Fig. 8.13 (a) and (b) respectively. In SM wave motion, the amount of wave energy trapped is found to be increasing with a decrease in the value of h_1/H (Fig. 8.13 (a)). On the other hand, in case of IM wave motion an opposite trend is observed, where the wave trapping is found to be increasing with an increase in the value of h_1/H (Fig. 8.13 (b)).

The influence of depth ratio h/H on the reflection coefficients in SM and IM are shown in Fig. 8.14 (a) and (b) respectively. Highest wave trapping in SM wave motion is observed when interface is located in between the free surface and breakwater free edge (Fig. 8.14 (a)). On the other hand, similar to the case of bottom-standing breakwater, highest wave trapping is observed in IM wave motion when the interface is located just at the center of free surface and the seabed (Fig. 8.14 (b)).

The reflection coefficients versus the normalized breakwater position L/λ_j for ($j = I, II$) are plotted in SM and IM for various values of s in Fig. 8.15 (a) and (b) respectively. In Fig. 8.15 (a), wave trapping in SM is found to be higher for smaller values of s . However, it is observed that the value of wave reflection in SM is same for $s = 0.5$ and 0.25 . On the other hand, the reflection coefficient in IM is found to be high for the intermediate values of s ($= 0.5$) (Fig. 8.15 (b)).

Breakwater Deflection and Hydrodynamic Force

The surface-piercing breakwater deflection and hydrodynamic forces are plotted in Figs. 8.16 and 8.17. The breakwater deflection is increasing with the increase in the value of G (Fig. 8.16 (a)) and h_1/H (Fig. 8.16 (b)). In Fig. 8.17 (a) the hydrodynamic force is increasing with the increase in the value of h_1/H . However, the hydrodynamic forces attain a constant value for all h_1/H values at higher value of breakwater flexural rigidity. It is expected as at higher values of breakwater rigidity the breakwater starts behaving like a rigid wall. On the other hand, the hydrodynamic force is high for small and large values of s (Fig. 8.17 (b)).

8.6.3 Summary of Important Observations

In both the cases of bottom-standing and surface-piercing breakwaters, reflection coefficients in SM and IM are periodic in nature for normalized breakwater positions. Full reflection takes place at $L/\lambda_j = n/2$, for $n = 1, 2, \dots, j = I, II$ and minimum reflection occurs at an intermediate points of $(n - 1)/2 < L/\lambda_j < n/2$, for $n = 1, 2, \dots$ and ($j = I, II$). A similar observation is reported in single-layer fluid (see Sahoo et al. (2000) and Yip et al. (2002)). The important observations from the present numerical results for wave trapping by porous partial breakwaters are summarized pointwise as below:

The Case of a Bottom-Standing Breakwater

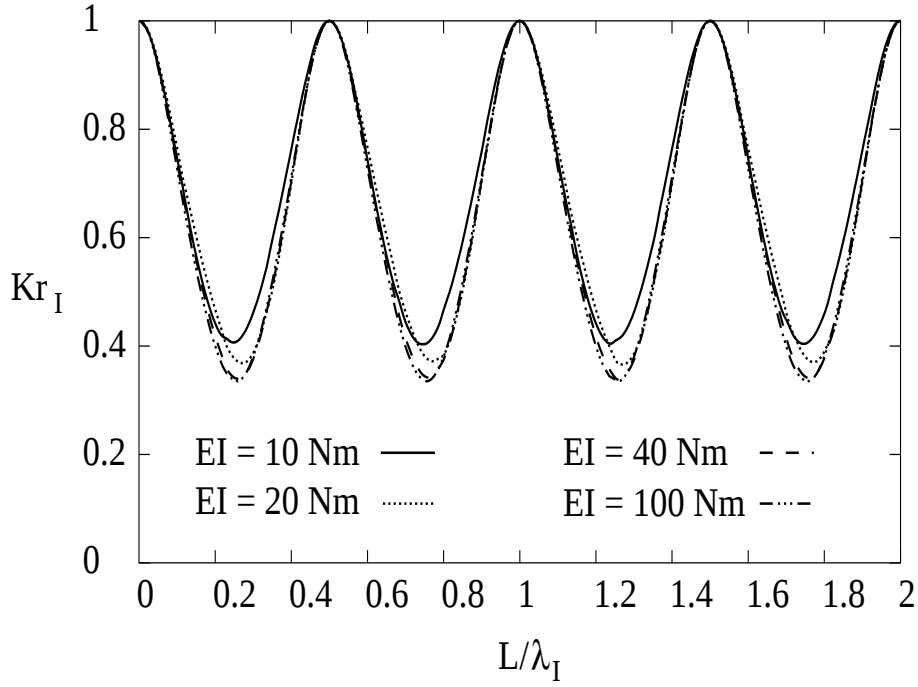
1. High wave trapping for waves in SM is observed for higher values of EI and the influence of EI on wave motion in IM is insignificant.

2. The amount of trapped waves in SM is increasing with a decrease in the porous-effect parameter and highest wave trapping in IM is observed for moderate values of porous-effect parameters.
3. For $G = 1 + 1i$ and $G = 2$, the minimum values of the reflection coefficient in SM and IM are same in magnitude and the only difference is the breakwater location at which the minimum reflection coefficient is observed. This may be due to the change in phase angle of the porous-effect parameter.
4. Highest wave trapping in SM is observed when the breakwater length, b/H just exceeds the depth of the lower fluid. The change in breakwater length has negligible effect on trapping of waves in IM in the region between the breakwater and the channel end-wall.
5. Highest wave trapping is observed in SM wave motion when interface is nearer to the seabed and highest wave trapping is observed in IM wave motion when the interface is located just at the center of free surface and the seabed.
6. Wave trapping in SM is found to be increasing with the increase in the value of s and the wave trapping in IM is found to be decreasing with the increase in the value of s .
7. Except for $b/H = 0.8$, in general the breakwater deflection is decreasing with the decrease in the value of b/H .
8. Breakwater deflection is increasing with the decrease in the value of h/H .
9. Hydrodynamic force is decreasing with the increase in the value of G . However, higher hydrodynamic forces are observed for G with higher values of inertial effect of fluid inside the porous media, which is represented by the imaginary part of G .
10. Hydrodynamic force is increasing with the increase in the value of s .

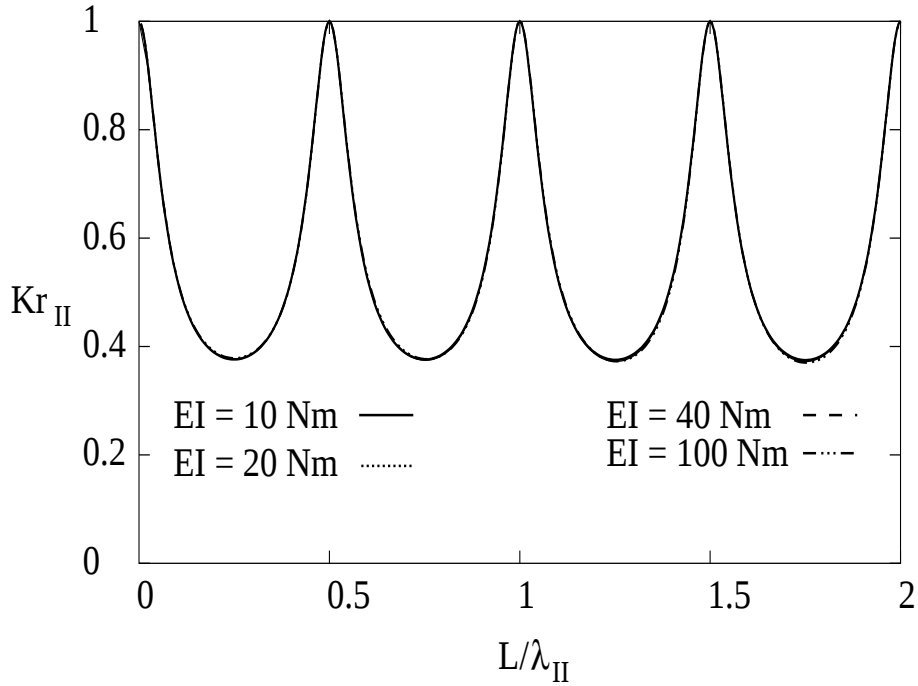
The Case of a Surface-Piercing Breakwater

1. High wave trapping in SM is observed for moderate value of breakwater flexural rigidity and in the case of wave motion in IM the maximum wave trapping is observed for high value of breakwater flexural rigidity.

2. The amount of trapped waves in SM is increasing with a decrease in the porous-effect parameter and in the case of wave motion in IM the amount of trapped waves is less for both small and large values of $|G|$.
3. The amount of wave energy trapped in SM is found to be increasing with an increase in $1 - b/H$ and wave trapping in IM is found to be high when the length of submergence of the breakwater is equal or greater than the depth of the upper fluid.
4. The amount of wave energy trapped in SM is found to be increasing with a decrease in the value of h_1/H and the wave trapping in IM is found to be increasing with an increase in the value of h_1/H .
5. Highest wave trapping in SM wave motion is observed when interface is located in between the free surface and breakwater free edge. Similar to the case of a bottom-standing breakwater, highest wave trapping is observed in IM wave motion when the interface is located just at the center of free surface and the seabed.
6. Wave trapping in SM is found to be higher for smaller values of s and the reflection coefficient in IM is found to high for the intermediate value of s .
7. Breakwater deflection is increasing with the increase in the value of G and h_1/H .
8. Hydrodynamic force is increasing with the increase in the value of h_1/H .
9. Hydrodynamic force is high for small and large values of s .

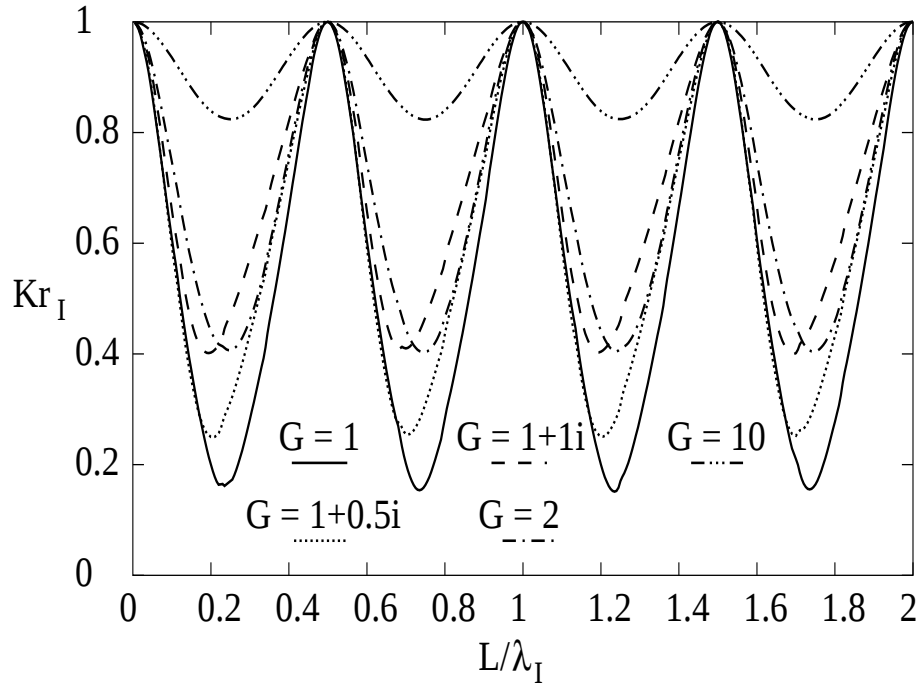


(a)

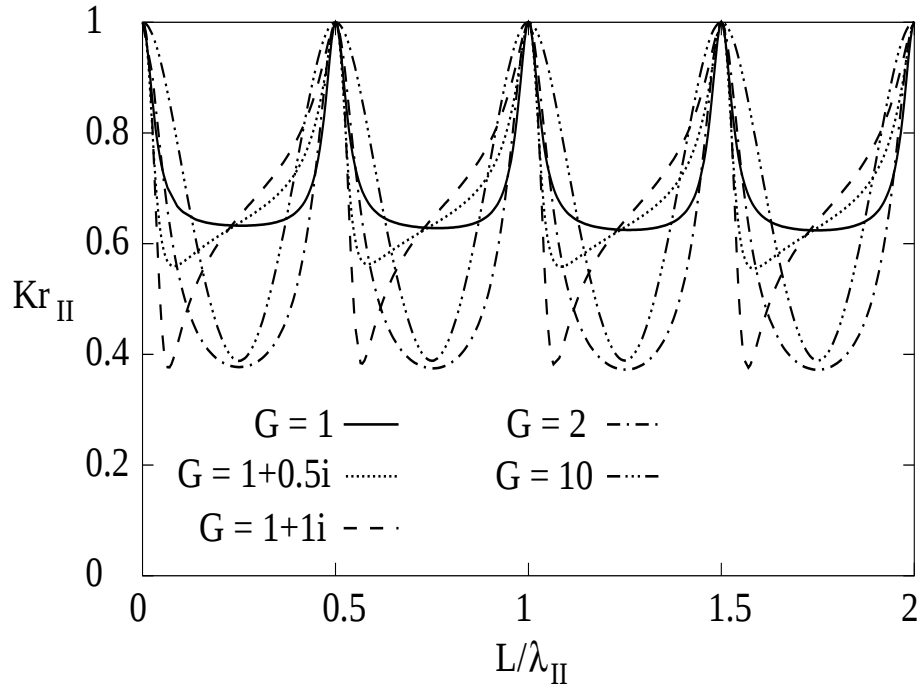


(b)

Figure 8.3: Reflection coefficients in (a) SM, Kr_I versus L/λ_n and (b) IM, Kr_{II} versus L/λ_{II} for bottom-standing breakwater at different EI values, $b/H = 1.0$, $G = 2$, $h/H = 0.25$, and $s = 0.75$.

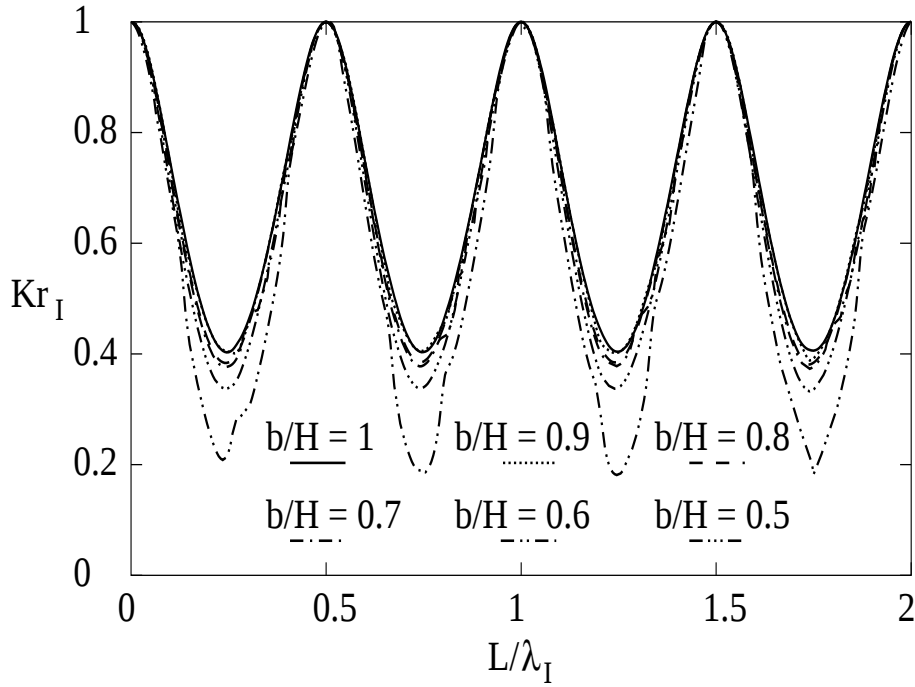


(a)

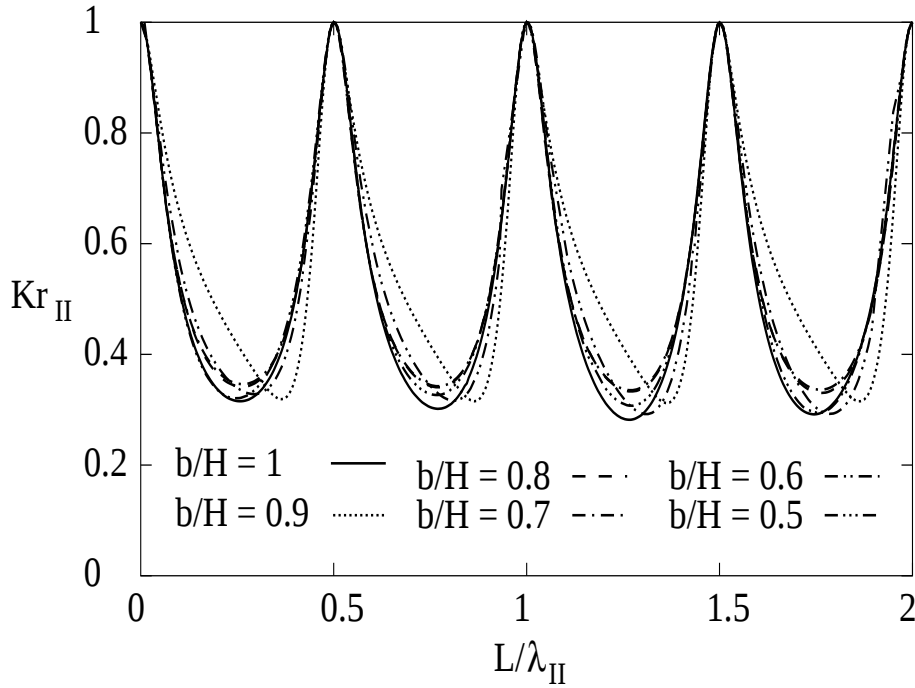


(b)

Figure 8.4: Reflection coefficients in (a) SM, Kr_I versus L/λ_n and (b) IM, Kr_{II} versus L/λ_{II} for bottom-standing breakwater at different G values, $h/H = 0.25$, $b/H = 1.0$, $EI = 10 \text{ Nm}$, and $s = 0.75$.

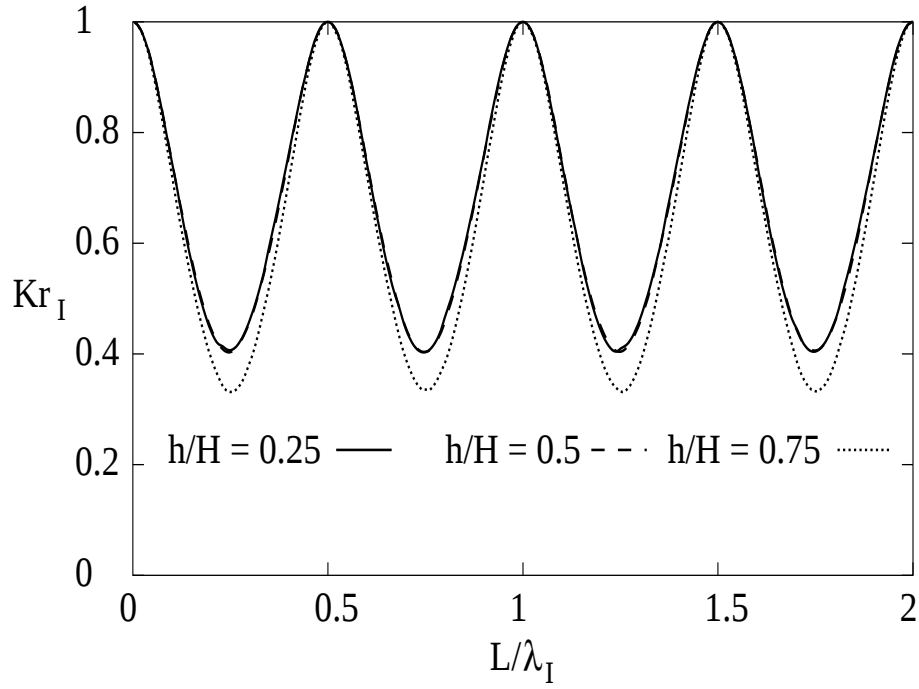


(a)

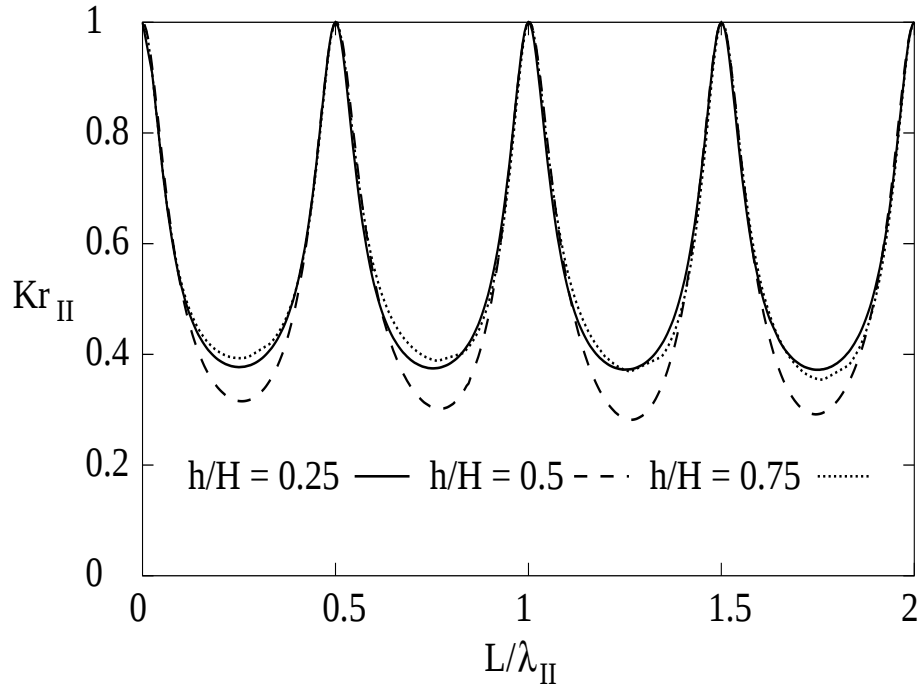


(b)

Figure 8.5: Reflection coefficients in (a) SM, Kr_I versus L/λ_n and (b) IM, Kr_{II} versus L/λ_{II} for bottom-standing breakwater at different b/H values, $h/H = 0.5$, $G = 2$, $EI = 10 \text{ Nm}$, and $s = 0.75$.

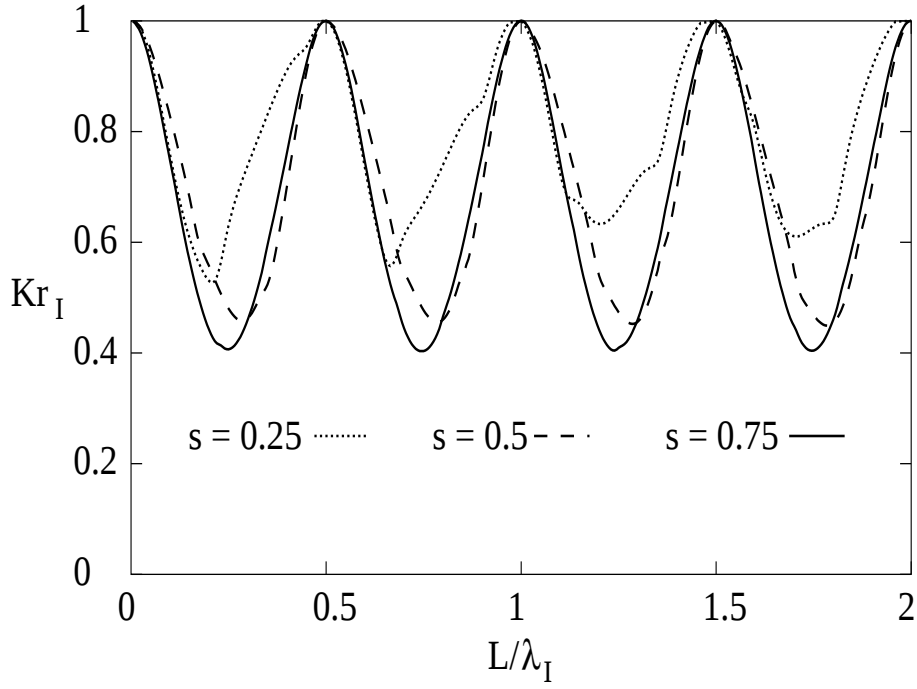


(a)

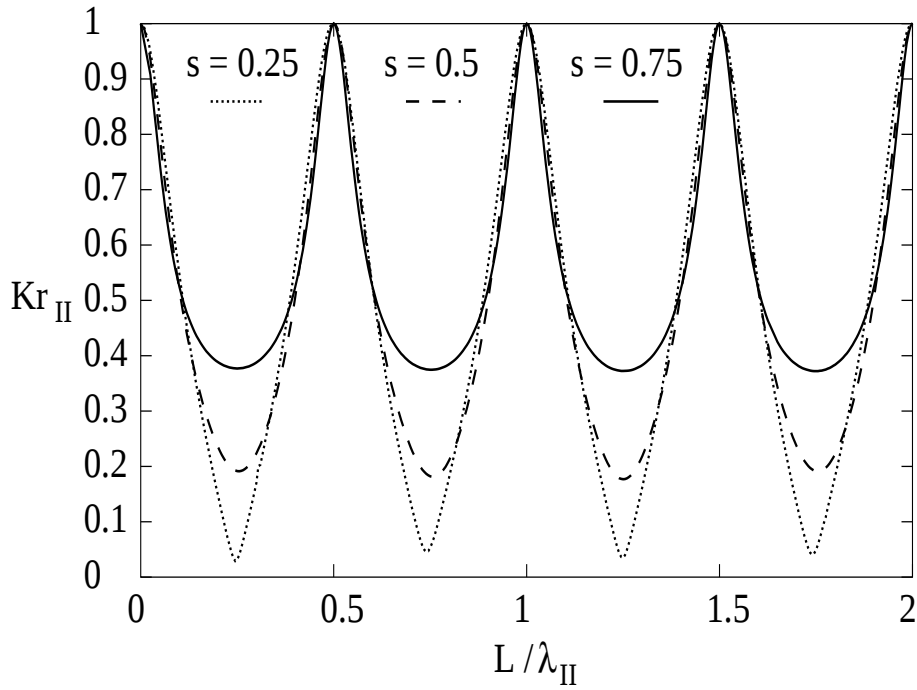


(b)

Figure 8.6: Reflection coefficients in (a) SM, Kr_I versus L/λ_n and (b) IM, Kr_{II} versus L/λ_{II} for bottom-standing breakwater at different h/H values, $b/H = 1.0$, $G = 2$, $EI = 10 \text{ Nm}$, and $s = 0.75$.

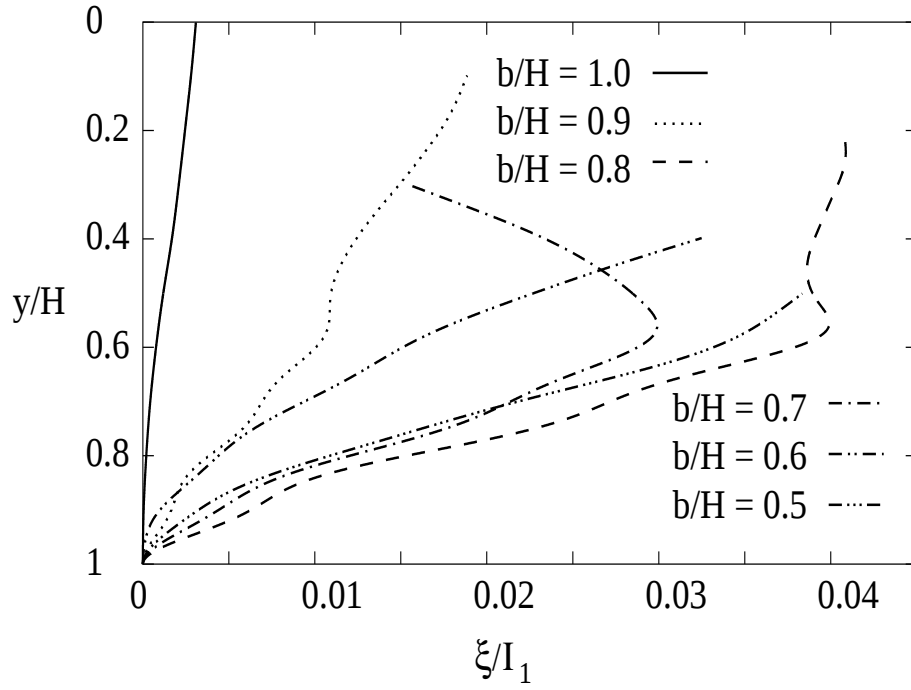


(a)

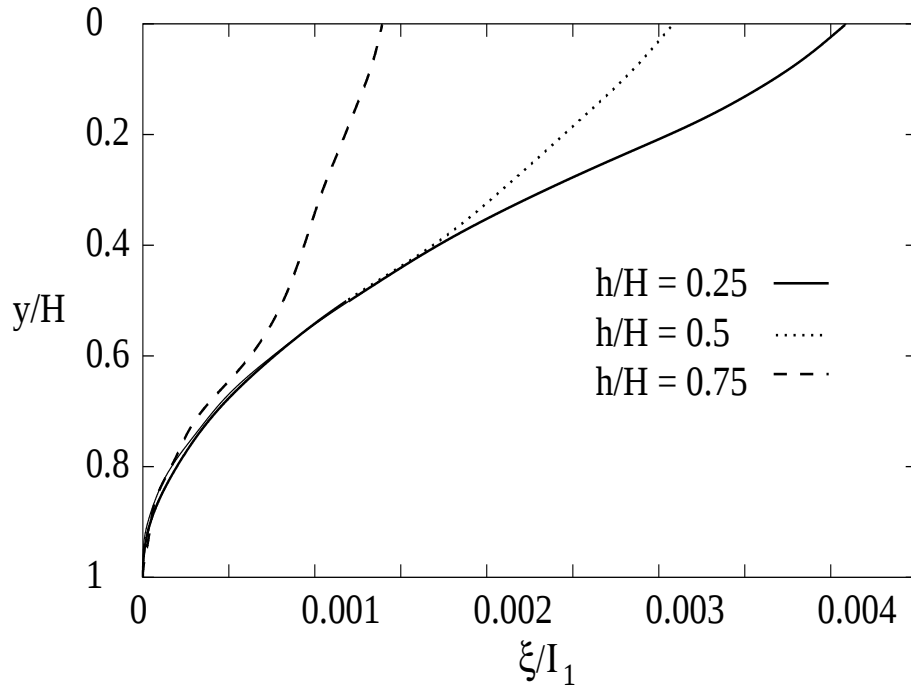


(b)

Figure 8.7: Reflection coefficients in (a) SM, Kr_I versus L/λ_n and (b) IM, Kr_{II} versus L/λ_{II} for bottom-standing breakwater at different s values, $h/H = 0.25$, $G = 2$, $EI = 10 \text{ Nm}$, and $b/H = 1.0$.

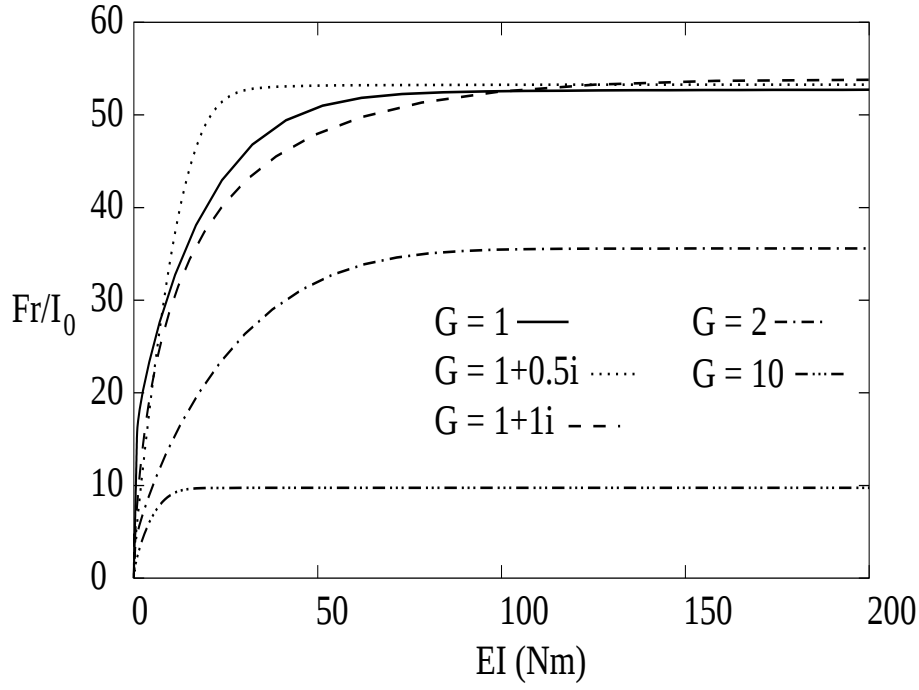


(a)

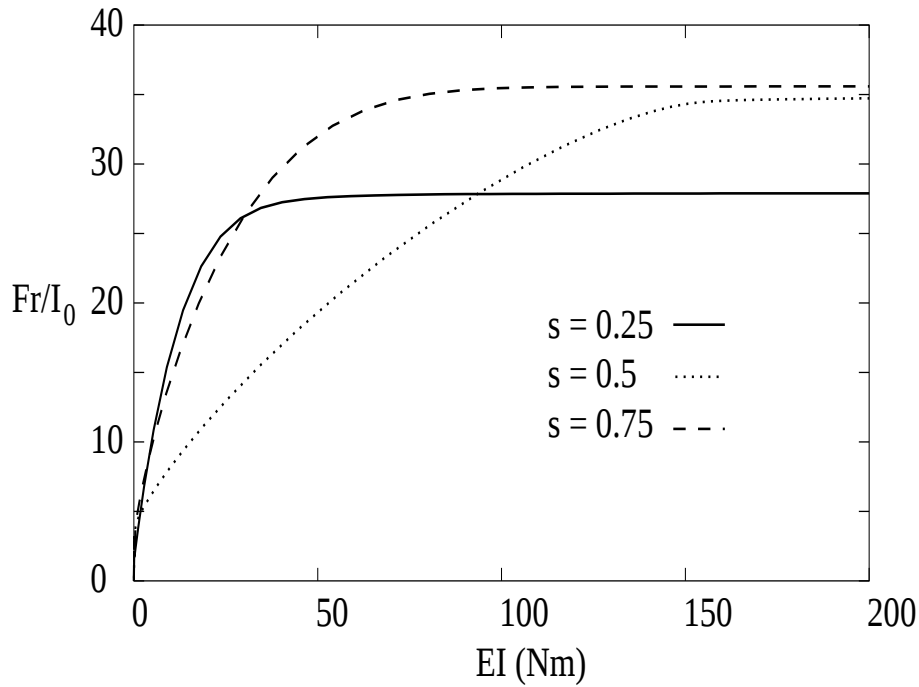


(b)

Figure 8.8: Barrier deflection, ξ at (a) $h/H = 0.25$ and (b) $b/H = 1.0$ for bottom-standing breakwater with $s = 0.75$, $G = 1$, $EI = 20 \text{ Nm}$, and $L/\lambda_I = 0.25$.

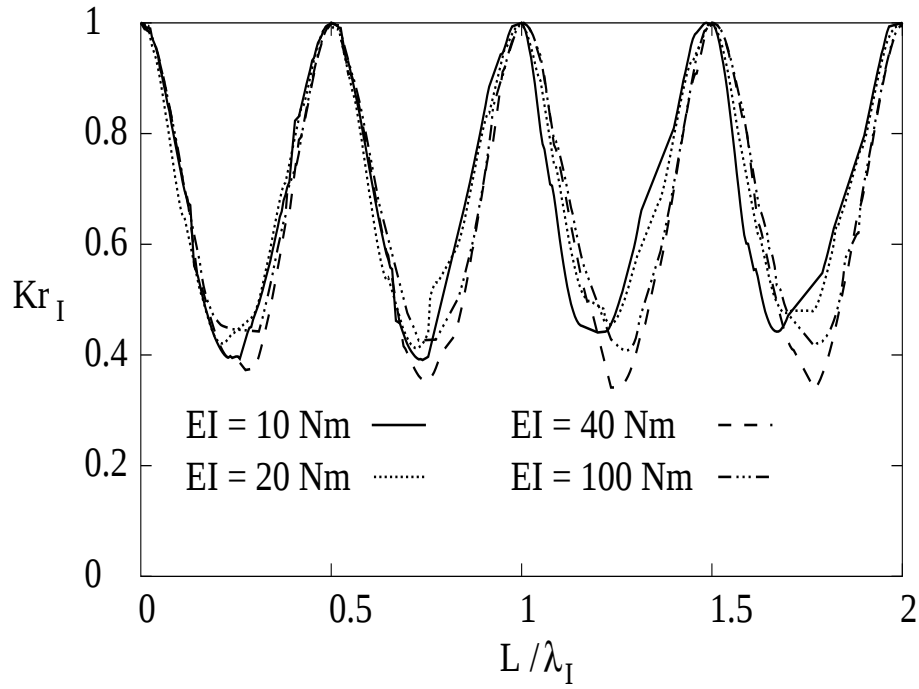


(a)

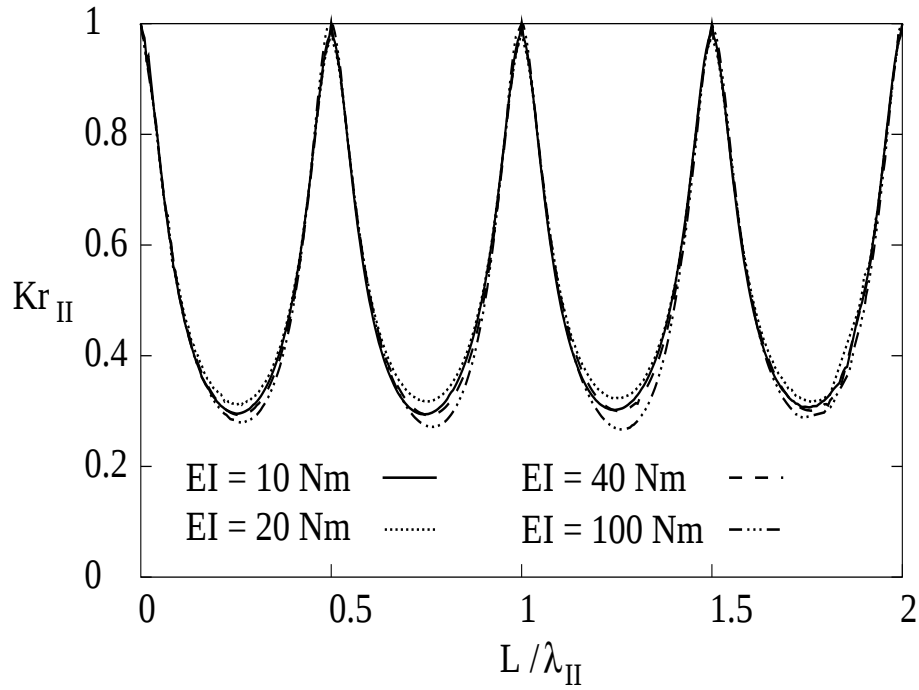


(b)

Figure 8.9: Hydrodynamic force, Fr at (a) $s = 0.75$ and (b) $G = 2$ for bottom-standing breakwater with $h/H = 0.25$, $b/H = 1.0$, and $L/\lambda_I = 0.25$.

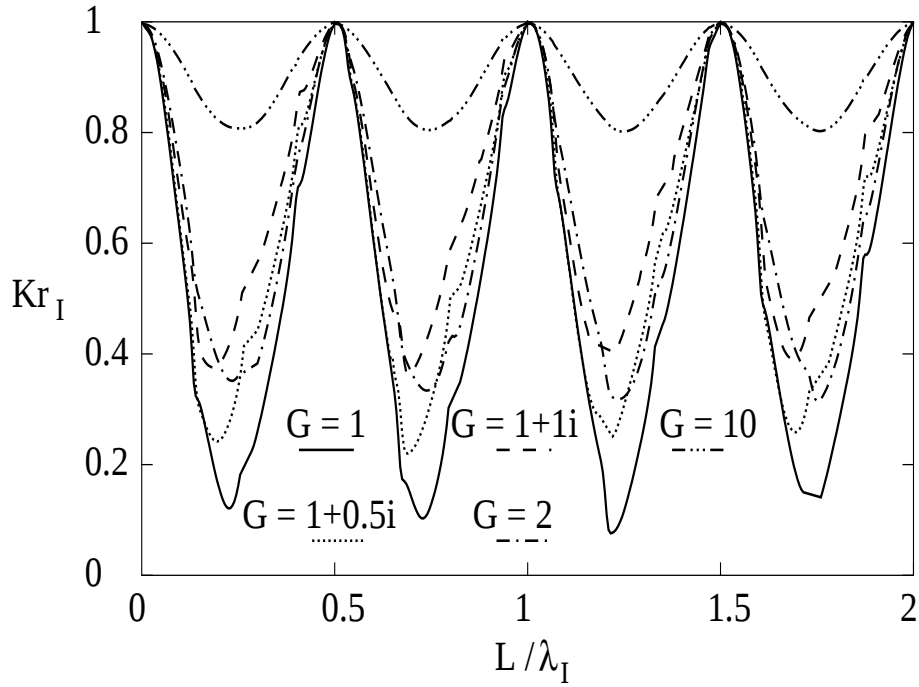


(a)

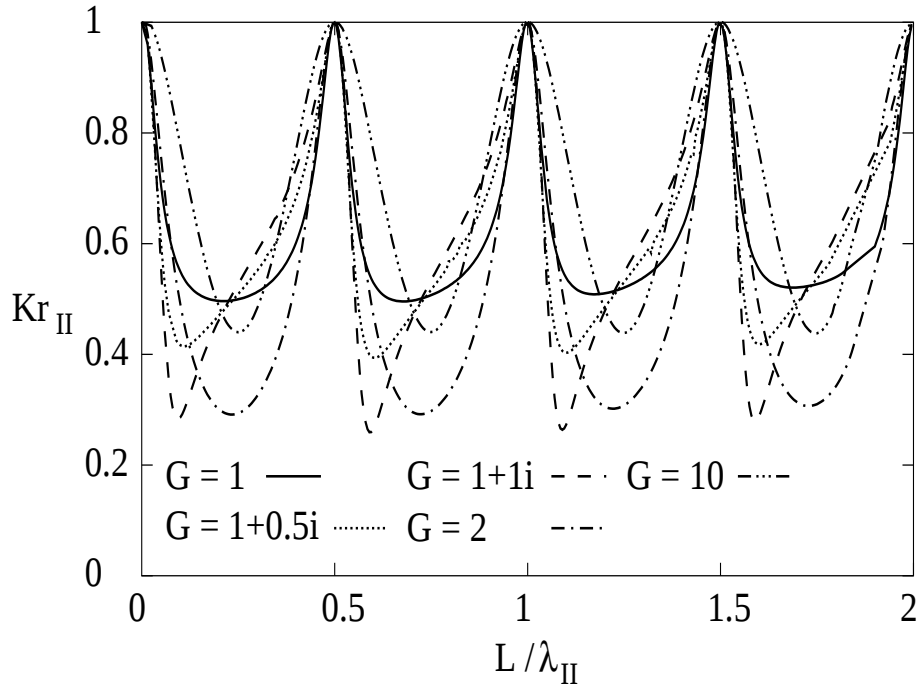


(b)

Figure 8.10: Reflection coefficients in (a) SM, Kr_I versus L/λ_n and (b) IM, Kr_{II} versus L/λ_{II} for surface-piercing barrier at different EI values, $b/H = 0.5$, $h_1/H = 1.0$, $G = 2$, $h/H = 0.5$, and $s = 0.75$.

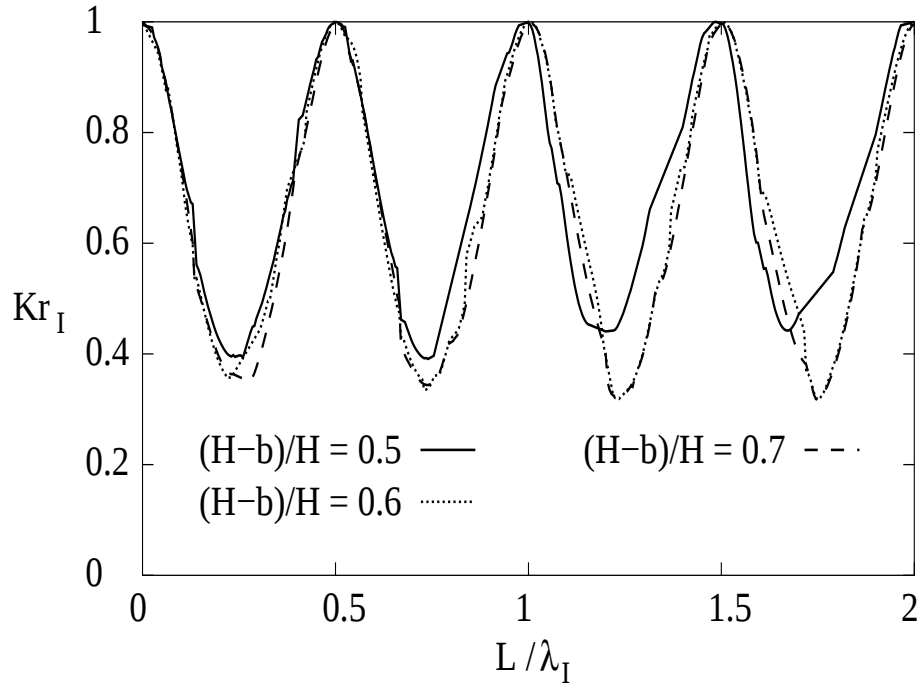


(a)

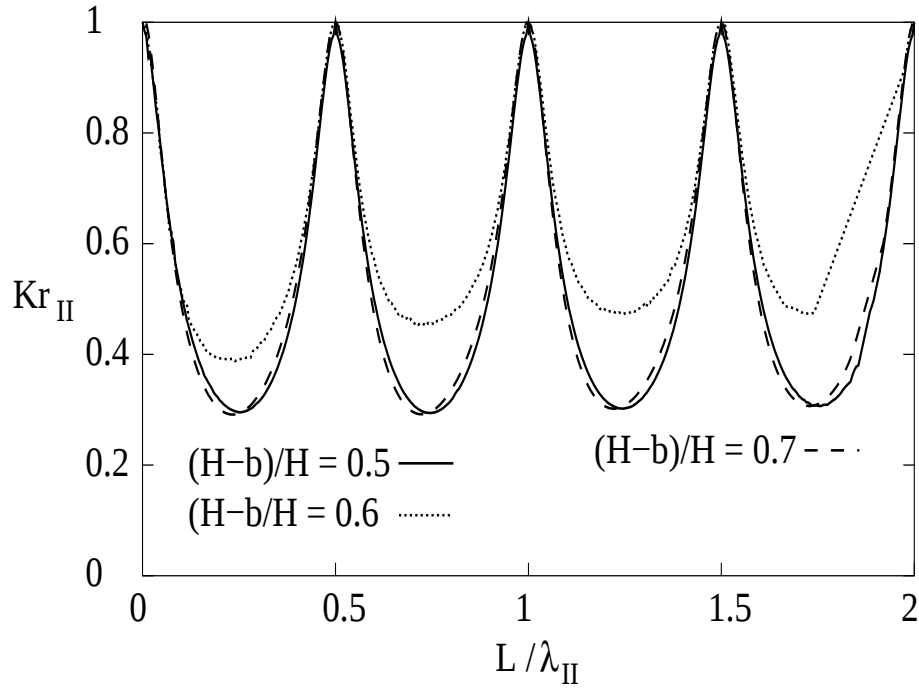


(b)

Figure 8.11: Reflection coefficients in (a) SM, Kr_I versus L/λ_n and (b) IM, Kr_{II} versus L/λ_{II} for surface-piercing barrier at different G values, $h/H = 0.5$, $b/H = 0.7$, $h_1/H = 1.0$, $EI = 10 \text{ Nm}$, and $s = 0.75$.

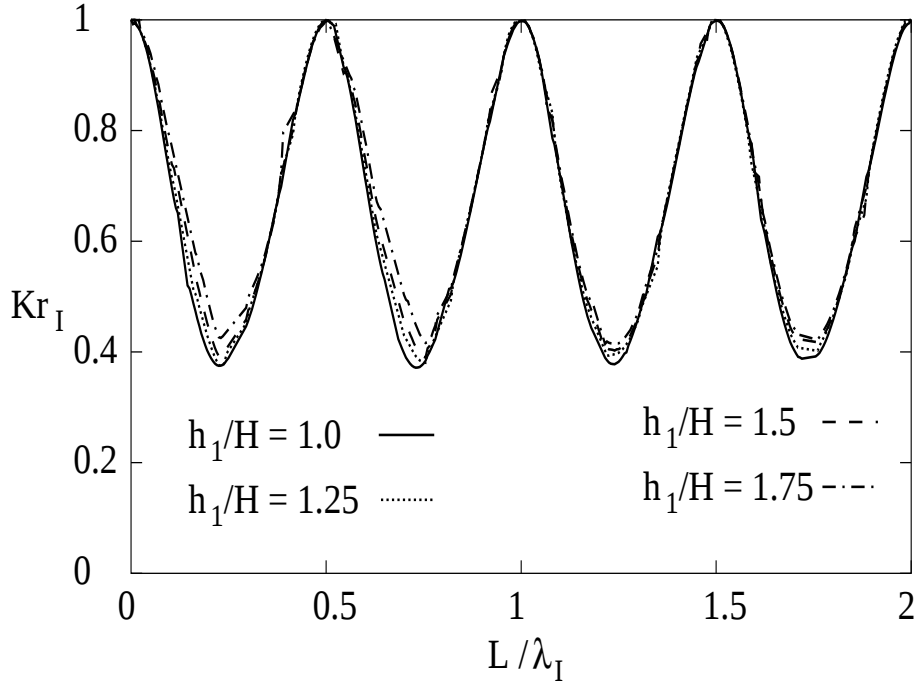


(a)

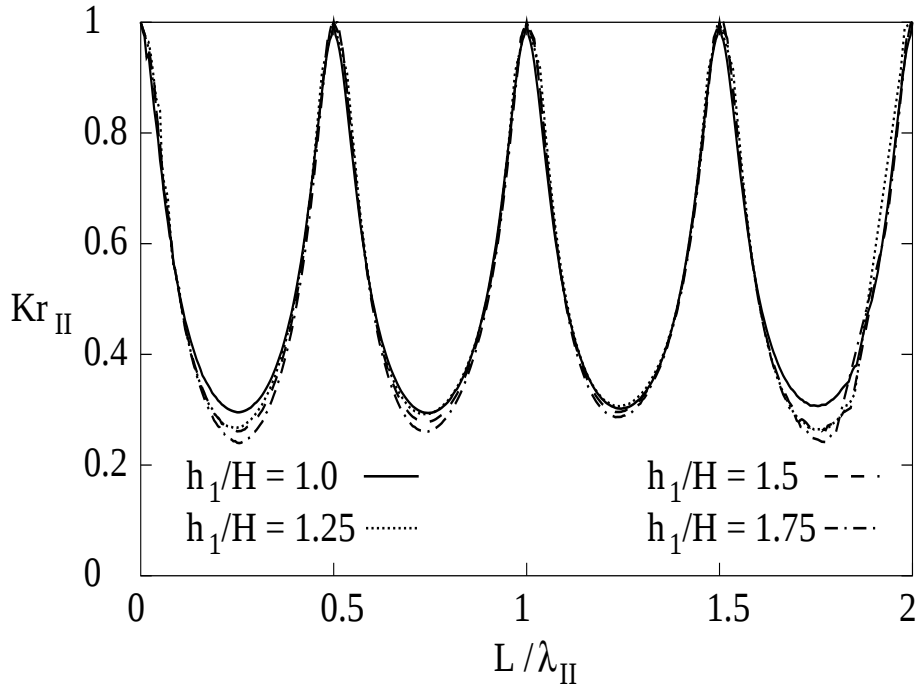


(b)

Figure 8.12: Reflection coefficients in (a) SM, Kr_I versus L/λ_n and (b) IM, Kr_{II} versus L/λ_{II} for surface-piercing barrier at different b/H values, $h/H = 0.5$, $G = 2$, $h_1/H = 1.0$, $EI = 10 \text{ Nm}$, and $s = 0.75$.

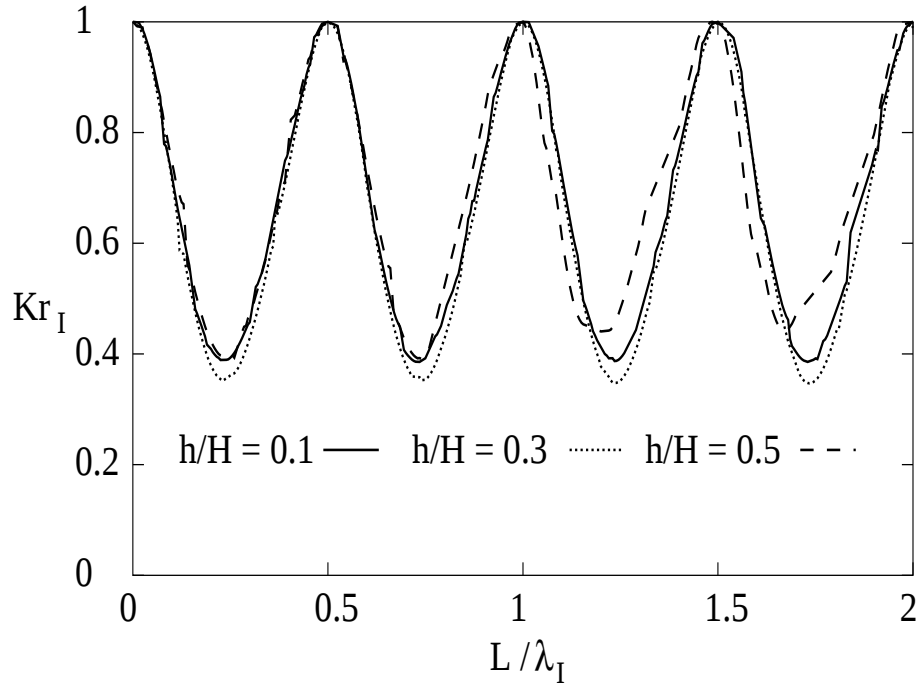


(a)

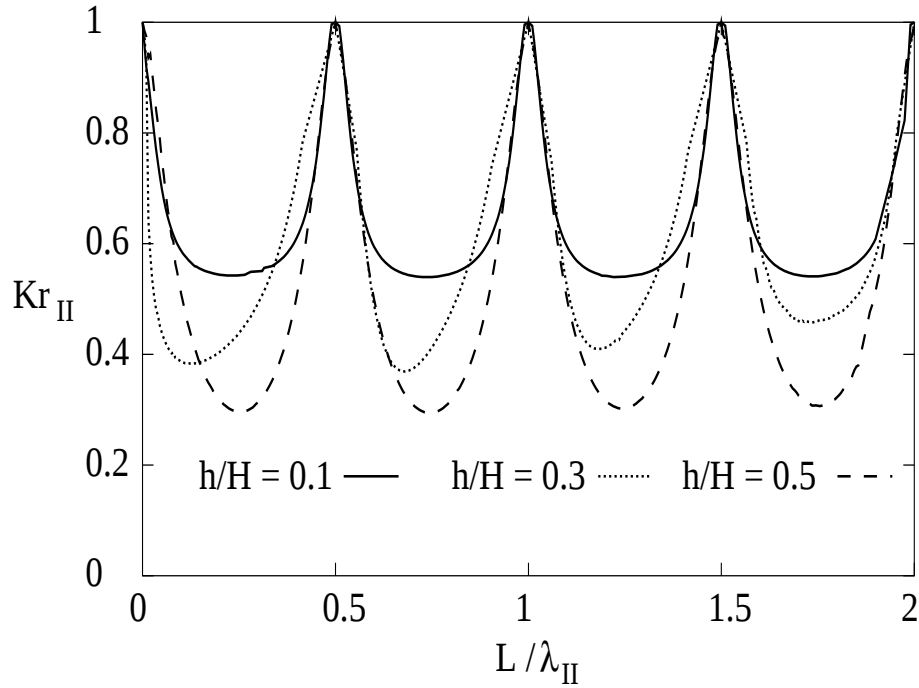


(b)

Figure 8.13: Reflection coefficients in (a) SM, Kr_I versus L/λ_n and (b) IM, Kr_{II} versus L/λ_{II} for surface-piercing barrier at different h_1/H values, $h/H = 0.5$, $G = 2$, $b/H = 0.5$, $EI = 10 \text{ Nm}$, and $s = 0.75$.

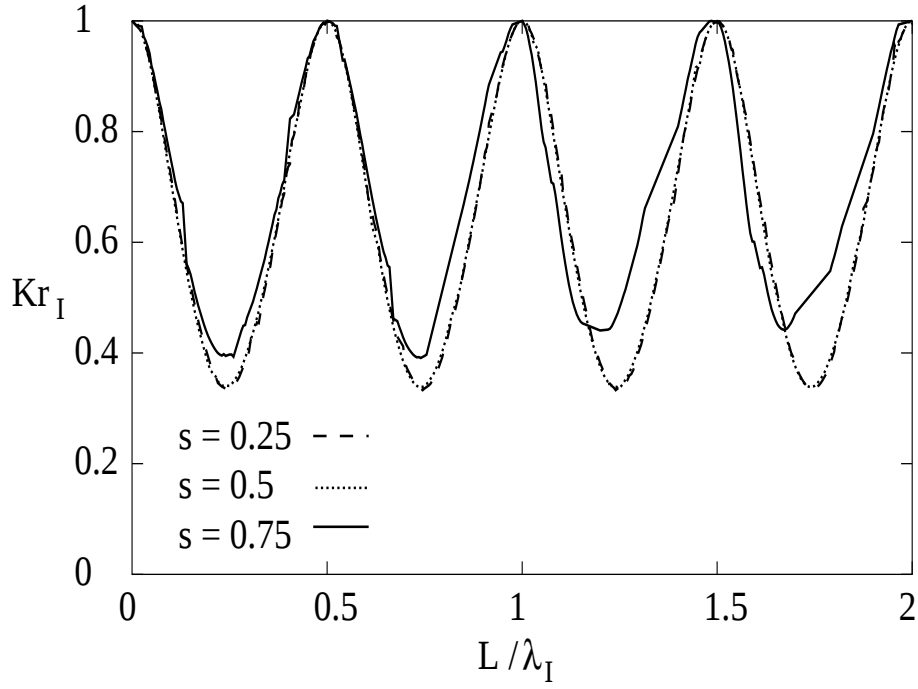


(a)

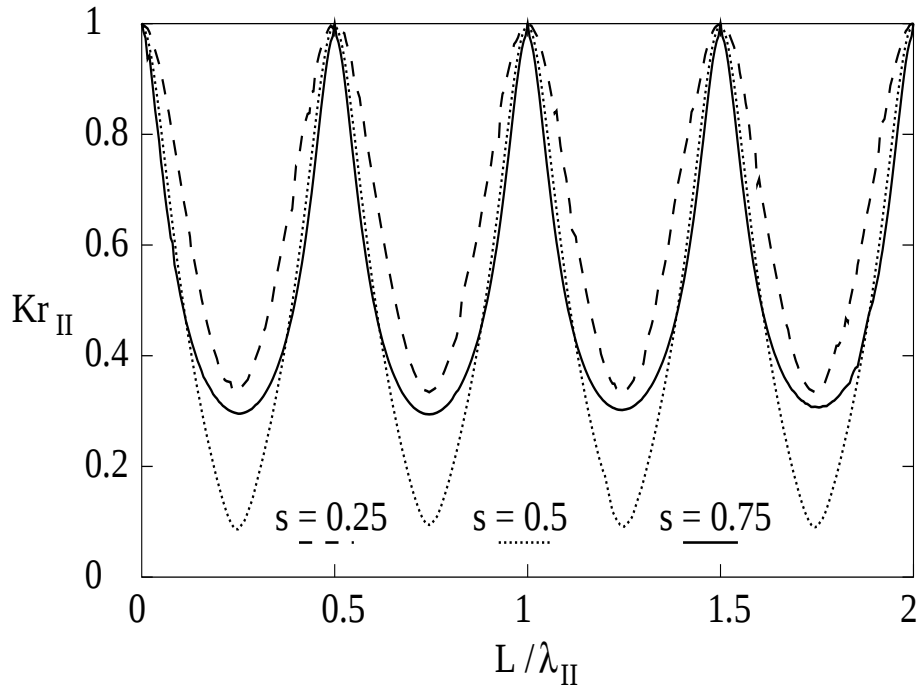


(b)

Figure 8.14: Reflection coefficients in (a) SM, Kr_I versus L/λ_n and (b) IM, Kr_{II} versus L/λ_{II} for surface-piercing barrier at different h/H values, $b/H = 0.5$, $h_1/H = 1.0$, $G = 2$, $EI = 10 \text{ Nm}$, and $s = 0.75$.

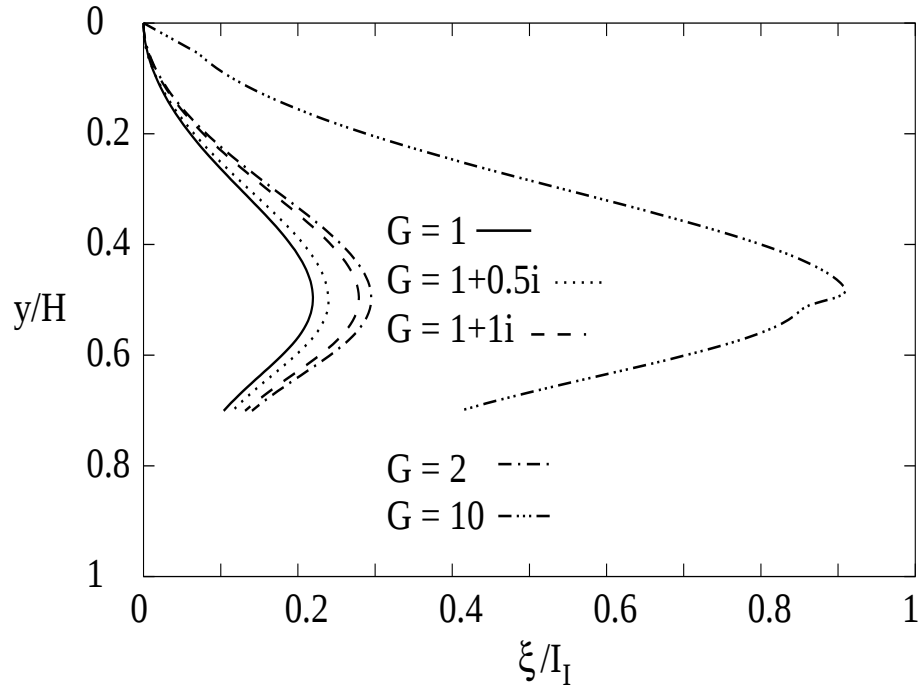


(a)

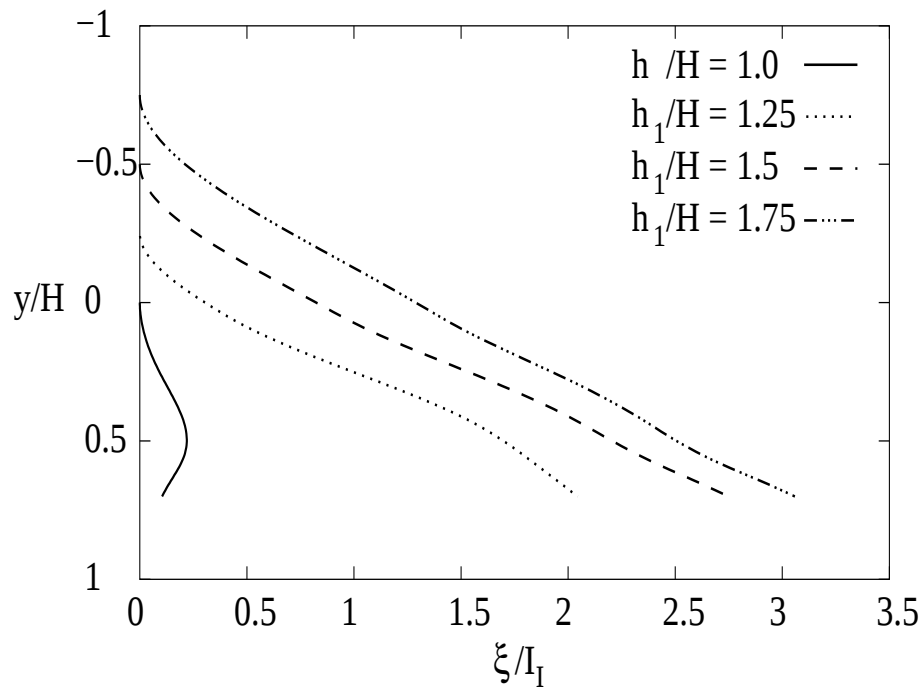


(b)

Figure 8.15: Reflection coefficients in (a) SM, Kr_I versus L/λ_n and (b) IM, Kr_{II} versus L/λ_{II} for surface-piercing barrier at different s values, $h/H = 0.25$, $G = 2$, $EI = 10$ Nm, $h_1/H = 1.0$ and $b/H = 0.5$.

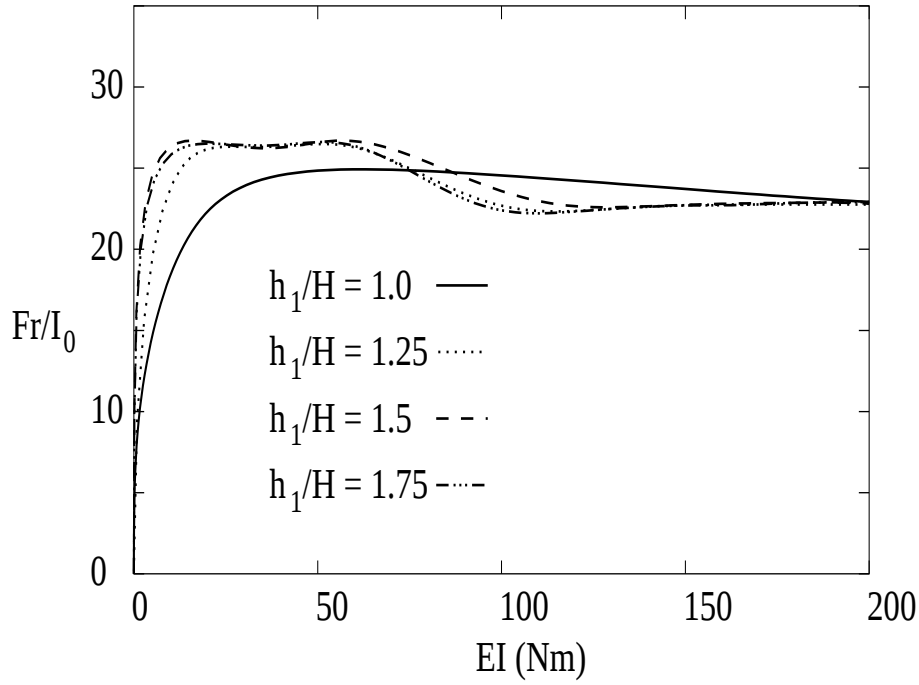


(a)

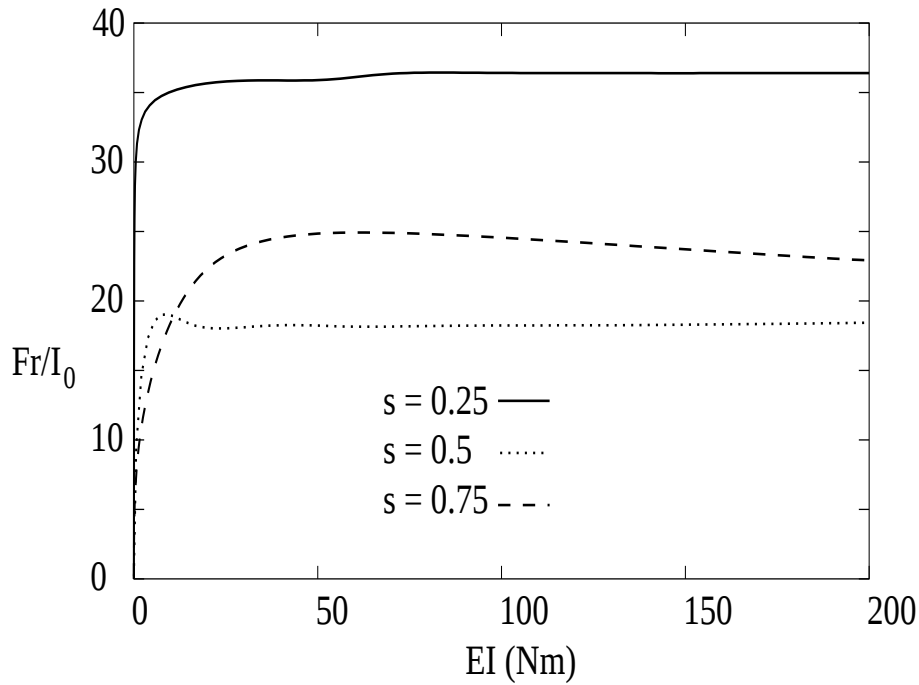


(b)

Figure 8.16: Barrier deflection, ξ at (a) $h_1/H = 1.0$ and (b) $G = 2$ for surface-piercing breakwater with $h/H = 0.5$, $b/H = 0.7$, $EI = 10 \text{ Nm}$, $s = 0.75$ and $L/\lambda_I = 0.25$.



(a)



(b)

Figure 8.17: Hydrodynamic force, Fr at (a) $s = 0.75$ and (b) $h_1/H = 1.0$ for surface-piercing breakwater with $h/H = 0.25$, $b/H = 0.7$, $G = 2$ and $L/\lambda_I = 0.25$.

Chapter 9

CONCLUDING REMARKS

9.1 SALIENT FEATURES OF THE STUDY

The salient features of the present research work are summarized as follows:

1. **Generalization of the Single-Layer Fluid Structure Interactive Models to Two-Layer Systems**

Models based on linearized-theory of water waves have been developed to generalize a class of 2-D wave-structure interaction problems in a single-layer fluid to a two-layer fluid. In a single-layer fluid a number of models have been developed over the years to solve wave-structure interaction problems. In the case of a two-layer fluid, time-harmonic waves can propagate with two different wave numbers, which makes the mathematical modeling and analysis difficult. In the present study it is assumed that the fluid is inviscid and incompressible, and the effect of surface tension is neglected. Special orthogonal/orthonormal relations suitable for the two-layer fluid are introduced to simplify the equations in the mathematical models. Computed results in the study are compared with the available results in the single-layer fluid. Furthermore, the numerical results from two different methods (matched-eigenfunction-expansion method and WSAM) are also compared with each other. The results show a close agreement when the structures are widely placed.

2. Investigation of Surface- and Internal-Wave Scattering

An attempt has been made to understand the scattering of surface- and internal-waves in a two-layer fluid. Scattering of water waves has been and continues to be a subject of much research. From the practical side, it is essential for many important ocean engineering problems to consider surface- and internal-waves together. From the research aspect, interest lies in understanding the complex flow physics. In the present work scattering of surface- and internal-waves by a single and a pair of identical rectangular dikes in a two-layer fluid is analyzed in two-dimensions within the context of linearized-theory of water waves. Both the cases of surface-piercing and bottom-standing dikes are considered. The scattering analysis for surface- and internal-waves has been extended for the wave past flexible porous structures, namely (i) Flexible porous membrane and (ii) Flexible porous plate. The numerical results are generated and are analyzed to understand the effect of various physical parameters on scattering for surface- and internal-waves.

3. Evaluation of Efficiency of Flexible Porous Breakwaters

In connection with the design of a breakwater, the functional performance of and the environmental load on the structure under various incident wave conditions must be known a priori. A number of studies have been reported in the literature for evaluation of efficiency of rigid breakwaters. Due to the rapid developments in material science, flexible structures are believed to be extremely effective as breakwaters, absorbing or reflecting much of the wave energy. In resonance, wave energy must be radiated or dissipated so that the oscillation can be effectively suppressed. Recently porous breakwaters are proposed to tackle such difficult practical problems. In the present research work mathematical models are developed to investigate the efficiency of flexible porous breakwaters in a two-layer fluid. Analysis of surface- and internal-wave dissipation, structural response and hydrodynamic force on flexible porous breakwaters (porous membrane and plate) are considered for various physical parameters in a two-layer fluid. The important observations from the numerical

results are elaborated.

4. Study of Trapping Phenomena for Surface- and Internal-Waves

The study of wave trapping in various physical situations is important as it has applications in many coastal engineering applications like dynamics and sedimentology of the near shorezone through their interaction with ocean swells and surfs. In recent years, there is a tremendous interest in using partial breakwaters to control waves. Many researches in past studied the wave trapping phenomenon due to porous breakwaters at a finite distance from the end-wall. They observed that the wave reflection depends very much on the distance between the barrier and the end-wall. Moreover, to widen the range of wave trapping and reduce the wave force on the breakwater, flexibility is introduced on the breakwaters. In the present thesis, the trapping of surface- and internal-waves by porous and flexible partial breakwaters near the end of a semi-infinitely long channel is studied in a two-layer fluid of finite depth having a free surface- and an interface. In the study both surface-piercing and bottom-standing configurations are considered. The wave trapping for both surface- and internal-waves is analyzed for various physical parameters.

5. The Identification and Evaluation of Fluid Density Ratio and Interface Location as Two Major Physical Parameters Influencing Effectiveness of Breakwaters

One of the major objectives of present study is to investigate the influence of the fluid density ratio and interface location on the effectiveness of different breakwaters. It is observed and identified in the present study that the fluid density ratio and the interface location plays a vital role in the performance of the various breakwaters considered in the study. The influence of fluid density ratio and interface location on the different breakwater performance is described based on the observations from the numerical results generated from this thesis.

9.2 FUTURE SCOPE OF RESEARCH

This section discusses the possible extensions of the present investigation. They are presented below:

1. The immediate extension of the present research work is the three-dimensional and oblique water wave scattering in a two-layer fluid.
2. Present analysis can be extended to study the effect of current in a two-layer fluid.
3. Investigations can be made on the surface- and internal-wave scattering and trapping under a floating ice in a two-layer fluid.
4. The analysis of Very Large Floating Structures (VLFS) in a two-layer fluid is also an area which may be exploited.
5. Investigation of surface tension effect of water waves in the two-layer fluid can be an interesting extension of the present work.
6. Study of porous flexible wave maker problem in a two-layer fluid is an important area because of its immediate practical applications.
7. Though some work has been reported on high wave energy dissipating flexible wave chamber, no real investigation has been carried out in two-layer fluid wave motion. So there is a great scope to investigate the performance of high wave energy dissipating flexible wave chamber, in a two-layer fluid.
8. Development of a numerical scheme to analyze the wave interaction with floating and submerged structures of arbitrary geometry in a two-layer fluid using boundary integral equation method will be an interesting extension of the present investigation from the practical application point of view.

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LIST OF PUBLICATIONS FROM THE PRESENT THESIS WORK

The thesis comprises of the following contributory papers:

Peer Reviewed

1. **Suresh Kumar, P., and Sahoo, T., (2006). Wave interaction with a flexible porous breakwater in a two-layer fluid.** *Journal of Engineering Mechanics, ASCE*, 132 (9), 1007–1014.
2. **Suresh Kumar, P., Manam, S. R., and Sahoo, T., (2007). Wave scattering by flexible porous vertical membrane barrier in a two-layer fluid.** *Journal of Fluids and Structures*, 23 (4), 633–647.
3. **Suresh Kumar, P., Bhattacharjee, J., and Sahoo, T., (2007). Scattering of surface and internal waves by rectangular dikes.** *Journal of Offshore Mechanics and Arctic Engineering, ASME*, 129 (4), 306–317.

Manuscript in Preparation

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Mr. P. Suresh Kumar, born on 23rd June, 1978 is a bachelor. He is Indian by Nationality. Having passed the Higher Secondary (10+2) Examination of the Orissa Council of Higher Secondary Education, Bhubaneswar, in the first Division in 1995, he took his degree of B.E. in Mechanical Engineering with first class in the year 2000 from Utkal University, Orissa. He obtained his masters degree (M.Tech) with first class from Mechanical Engineering Department, Indian Institute of Technology, Guwahati with specialization in Fluid and Thermal Science in 2003. After post graduation he entered into the Doctoral Study in the Department of Ocean Engineering and Naval Architecture, at Indian Institute of Technology, Kharagpur in the year 2003. The author has had excellent opportunity to carry out research in a variety of areas in the fluid and thermal science during his graduate studies, including numerical modeling of flow past porous and flexible structures; wave motion in a two-layer fluid; scattering of water waves; internal waves; wave trapping; fluid structure interaction; flow physics in large-hydraulic-diameter ducts; heat transfer augmentation in heat exchangers; and, design and development of Savonius wind turbine blades. He has to his credit some publication in the aforementioned areas of fluid and

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LIST OF PUBLICATIONS OUTSIDE THE PRESENT THESIS WORK

It may be noted that publications from the present thesis work can be found separately as (LIST OF PUBLICATIONS FROM THE PRESENT THESIS WORK) in the thesis. The publications of the author out side the present thesis work are listed below.

Peer Reviewed

1. **Suresh Kumar, P.**, Oh, Y. M., and Cho, W. C., (2008). Surface and internal waves scattering by partial barriers in a two-layer fluid. *Journal of Korean Society of Coastal and Ocean Engineers*, 20(1), 25–33.

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